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Meaning, Context and Logical Truth

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Synopsis

Formal semantics, today a self-standing discipline, emerged at the interface of logic and philosophy of language in the early 1970’s, as a result of using the tools provided by logic to study natural language. In this paper, we start by offering a quick glance at those early developments, paying less attention to the historical aspects than to the philosophical motivations, in particular to truth-conditionality and compositionality. We then put forward a very basic model-theoretic apparatus for the treatment of intensional constructions (modal and temporal operators) as well as a class of context-dependent constructions (indexicals), following the analysis provided by David Kaplan in the 1970’s and widely endorsed nowadays. There is a great variety of issues that have to do with context-dependence and its impact on truth-conditional approaches to natural language, some thoroughly studied ever since (e.g. presupposition, anaphora), others having emerged more recently (e.g. gradable adjectives, derogatory terms, etc.), a discussion of which would go beyond the scope of this paper. Instead, we will focus on the interaction between context-dependence and logical truth. For example, the sentence “This red flower is red” contains the demonstrative ‘this’ and is context-dependent. Yet, if someone says this pointing to a red flower, not only will they say something true, but it seems that the truth thereby expressed is already secured by the meaning of the sentence (and the conditions under which it can be felicitously uttered). Could, then, this truth be a matter of logic? To take another example, if someone says “It is snowing”, can we infer “It is actually snowing here and now”? And, if inference there is, will it be a logical inference? The aim of the present paper is not to provide firm answers to such questions, but rather, to point out issues that arise when we think of logical truth as
truth in virtue of meaning, and when we try to reconcile this idea with the fact that the use of language is highly context-dependent.

1 The model-theoretic approach to meaning

It is often believed that Gottlob Frege (1848-1925) is the “father” of modern logic as well as of philosophy of language.1 But it was arguably Richard Montague (1930-1971) who set out to propose a systematized formal approach to natural language. The first two thirds of the 20th century (or equivalently, the time that elapsed since Frege’s work until Montague’s work) were nevertheless marked by significant progress in the realm of formal logic, and model-theory in particular. At the time at which Montague and his contemporaries, like David Lewis and Max Cresswell, started laying down the foundations for natural language semantics, the semantics for formal languages had already been in place, and well-developed. The title of one of Montagues’ pioneering paper, English as a Formal Language, aptly captures precisely the gist of Montague’s approach. Until then, it was a widespread conviction, as much among philosophers or logicians as among linguists, that there is an unbridgeable gap between, on the one hand, the language of logic, mathematics, and possibly science, whose sentences, or formulae, can be given precise truth conditions and allow for formal and model-theoretic analyses, and, on the other, natural languages such as English, Chinese, or Bengali, which were believed to be too hectic, too “imperfect”, to be formally tractable.2 The gist of the Montagovian program and, in that respect, of the field of formal semantics qua a discipline of linguistics, is to challenge this conviction, and to apply the same sort of analysis that had been made available for formal languages, to ordinary language.

One of the central tenets of natural language semantics, and, more gen-

1For logic, the most relevant source is Frege’s Begriffsschrift (Frege 1879), while for philosophy of language, it is Uber Sinn und Bedeutung (Frege 1892). On Frege being perceived as the founder of logic and semantics, e.g. Heim and Kratzer 1998, a widely used introduction to formal semantic, write: “[Frege’s] work marked the beginning of both symbolic logic and the formal semantics of natural language” (p. 2).

2Frege himself was convinced of a deep cleavage between the two families of language. He wrote e.g.: “If our language were logically more perfect, we would perhaps have no further need of logic, or we might read it off from the language. But we are far from being in such a position. Work in logic just is, to a large extent, a struggle with the logical defects of language” (1915, p.323).
Generally, of philosophy of language of the analytic tradition, is the idea that meaning should be cashed out in terms of truth conditions. To illustrate the idea with a simple example, consider:

1. Annie owns a car.

What does it take to understand (1)? The basic idea on which truth-conditional approaches rely is that in order to understand (1), one ought to know what the world must be like for (1) to come out true. One need not know whether (1) is true—rather, one needs to know that (1) is true in those and only those situations in which Annie owns a car. Note that there can be a variety of such situations. Annie might own a red car or a white car; she might own a Ferrari or a Honda; she might own a car she never drives; she might own a dozen of cars. The meaning of (1) does not distinguish among those situations. All that is required to understand (1) is to know that some such situation—one in which there is a car owned by Annie—must obtain for (1) to be true.

The second equally central tenet of natural language semantics is the idea that the meaning of a complex expression E can be obtained from the meanings of the simple expressions of which E is composed, and the way they compose it. Note that I have stated the idea somewhat vaguely; this is because the issue itself of how the idea is to be formulated is under controversy. The assumption endorsed by truth-conditional semantics is, simply, that meaning is compositional, but the question of what exactly compositionality amounts to is, to a certain extent, an open question. For our purposes, we may leave the question open, and simply retain that when S is a complex sentence composed of simpler sentences $S_1...S_n$, the truth-values of $S_1...S_n$ (with respect to the appropriate parameters) help determine the truth-value of S (with respect to the same or to some other appropriately related parameters). We will shortly see what this idea amounts to.

Let us wrap up this introductory section with a rough, somewhat simplified outline of semantics for a fragment of a language like English. A semantic framework for a language (natural or formal) may be thought of as a system such that, if you give it a sentence, it will return a truth value. But of course, before it can return a truth value, it will ask for further inputs. First and foremost, it will ask for a structure of interpretation. In the case of first order logic (FOL), the structure of interpretation consists simply of a set of individuals U, the universe, and of a certain function.
the interpretation function, which takes non-logical symbols either to elements of U (when the symbol is an individual constant) or to subsets of U (when the symbol is a monadic predicate) or to sets of n-tuples of individuals from U (when it is an n-place predicate). For example, the sentence in (1) can already be formalized in FOL using the existential quantifier \( \exists \), a monadic predicate ‘car’, a two-place predicate ‘own’, and an individual constant ‘Annie’: \( \exists x (\text{car}(x) \land \text{owns}(\text{Annie}, x)) \). This formula will be true in those and only those structures in which the intersection of the interpretation of the monadic predicate ‘car’ and of the set of things to which the interpretation of the name ‘Annie’ is related by the interpretation of the two-place predicate ‘own’ is not empty.

Now, even in FOL, the system sometimes cannot return a truth value even when we give it a structure of interpretation. That happens when the input sentence contains free variables. And that is presumably what will happen, too, if we gave the semantic system for natural language a sentence that contains (unbound) pronouns. Consider:

2 She owns a car.

Relative to one and the same structure of interpretation, (2) may be true and false. For instance, let “our” structure (one that adequately interprets English and is based on the actual state of affairs) be such that Annie owns a car while Betty doesn’t. Then (2) will be true with respect to the assignment function that assigns Annie to the free variable that stands for ‘she’, and false with respect to the one that assigns Betty to the same variable.

Crucially, FOL is unable to deal with intensional modal expressions such as ‘it is possible that’ and ‘it is necessary that’. Nor can it deal with intensional temporal expressions such as ‘it has been the case that’ or ‘it will be the case that’. Reconsider (1) and suppose that our world is one in which Annie owns no car. We want to say that (1), while false when evaluated with respect to our own world, is true when evaluated with respect to some possible world in which she owns a car. Similarly, suppose that Annie doesn’t own a car yet, but then gets one in 2015. Then we want to say that (1) is false if evaluated with respect to year 2010, but true if evaluated with respect to 2015. Now consider:

3 It might have been the case that Annie owned a car.

4 It will be the case that Annie owns a car.

We also want to say that (3), evaluated with respect to our world, the actual world, is true, because there is a possible world in which the sentence
in (1) is true, and because the sentence in (1) is, syntactically, a constituent of the more complex sentence in (3), in which it is embedded under the possibility operator. Similarly, we want to say that (4), evaluated with respect to the present time, is true, because there is a later time at which (1), embedded under the temporal operator in (4), is true.

Structures of interpretation for languages that contain intensional modal and temporal operators will thus specify not only a universe of individuals, but also a set of possible worlds (together with an “accessibility” relation on that set) and a set of times (together with an ordering “earlier-later” relation on that set). The interpretation function will now send an n-place predicate not just to a set of n-tuples of individuals from U, but to a mapping from world-time pairs to such sets. So, if we now give the system a sentence, expecting a truth value in return, the system will ask for a structure of interpretation, an assignment function, as well as a world of evaluation and a time of evaluation. The following notation means that sentence \( \varphi \) is true with respect to structure S, world w, time t, and assignment f of values to the free variables:

\[
[[\varphi]]_{S}^{w,t,f} = T
\]

So, for instance, the recursive truth definition of the possibility operator \( \Diamond \) as in (3) can be expressed in the following way:

\[
[[\Diamond \varphi]]_{S}^{w,t,f} = T \text{ iff there is } w' \text{ accessible from } w \text{ such that } [[\varphi]]_{S}^{w',t,f} = T
\]

Similarly, the recursive truth definition of the temporal ‘later than’ operator \( FUT \) as in (4) can be expressed as follows:

\[
[[FUT \varphi]]_{S}^{w,t,f} = T \text{ iff there is } t' \text{ later than } t \text{ such that } [[\varphi]]_{S}^{w,t',f} = T
\]

These definitions illustrate both truth-conditionality and compositionality. The idea that meaning is compositional is reflected through the fact that the meaning of a complex sentence of the form ‘it might have been the case that \( \varphi \)’ is a function of the meaning of the operator ‘it might have been the case that’ and of \( \varphi \) (and similarly, mutatis mutandis, for the operator ‘it will be the case that’). The truth-conditionality is, in turn, reflected through the fact that the meaning of the operator ‘it might have been the case that’ tells you under which conditions the complex sentence is true, given the conditions under which the embedded sentence is true.
2 Making room for context-dependence in truth-conditional semantics

It was believed for a long time that context-dependent expressions cannot, and should not, be part of any language for which a rigorous, truth-conditional analysis is to be provided. But since the 1970’s, considerable attention has been paid to such context-dependent constructions, and to the possible ways of incorporating them into formal frameworks. Hans Kamp (1971) and Frank Vlach (1973) were among the first to realize that the adverb ‘now’ posed a problem for tense logic, or, more precisely, those frameworks of tense logic that deploy a single time coordinate (as does the framework sketched in the previous section). To give you a flavor of the problem, consider this pair of sentences:

5 Maria said that she would call.

6 Maria said that she would call now.

The contrast between (5) and (6) shows that the word ‘now’ in English is not vacuous. For (5) to be true, there needs to be some time in the past at which Maria said that at some future time she would call; this future time can be some time or any other, it doesn’t matter. On the other hand, for (6) to be true, there needs to be some time in the past at which Maria said that at some specific future time she would call; and this time is no other than the one at which (6) is being said and evaluated for truth.\(^4\)

As another example that similarly requires amending the simple, standard sort of framework that deploys a single time coordinate, consider

7 Someday everything that is flourishing will be faded.

The sentence in (7) is ambiguous. On one reading its “non-charitable” reading it says that at some point in future, everything that is flourishing

\(^4\)It should be noted that while Kamp argues that the indexical ‘now’ cannot be straightforwardly handled within tense logic, he explicitly notes that if we use variables for times, then we should be able to capture the relevant sense of (6). He writes: “I of course exclude the possibility of symbolizing the sentence by means of explicit quantification over moments. (...) Such symbolizations, however, are a considerable departure from the actual form of the original sentences which they represent - which is unsatisfactory if we want to gain insight into the semantics of English.” (1971: 231f)

\(^5\)A similar example is discussed by Crossley and Humberstone (1977), concerning modal logic and the modal indexical ‘actually’, rather than tense logic and the temporal indexical ‘now’.
then, will be faded then. This reading is immediate in standard tense logic. On another reading, (7) says that if something is flourishing (now), then at some point in future, that thing will be faded. Now, one might point out that this reading, too, can be handled in standard tense logic, namely, by tampering with the scope that the quantifier and the temporal operator respectively take. The tentative formal rendering of (7) could go as follows:

8 \forall x (\text{flourishing}(x) \rightarrow \text{FUT}(\text{faded}(x)))

The cost, though, of proposing (8) as a way of formalizing (7) is that it involves, as Kamp would say, “a considerable departure” from the syntax of (7). But even if one is prepared to close one’s eyes on issues related to the syntax/semantics interface, there is arguably a lingering, purely semantic problem. Namely, there is yet a third reading that simple tense logic cannot account for, viz. the “collective” reading on which (7) says that there is some point in the future at which is faded everything that is flourishing now; everything is faded together, if you wish. This third reading entails (8), but note that (8) may be true even if there is no unique time at which everything now flourishing is faded. Imagine that right now, the begonias as well as the hortensias are flourishing, but that from now on, whenever the begonias flourish, the hortensias fade, and when the begonias fade, the hortensias flourish, and so on ad infinitum. In that case, the formula in (8) will come out true while (7) itself, under the reading under consideration, ought to come out false.

The gist of these examples, and similar examples involving the modal indexical ‘actually’, is to show that standard modal tense logic, with a single time coordinate and a single possible world coordinate, is not rich and powerful enough to handle certain natural language constructions. What is more, not only do the overt indexicals such as ‘now’ pose a problem. For, reconsider (3): the problematic construction is the present tense itself in ‘is flourishing’.

The solution to the problem put forward in early 70’s, the “double-indexing” solution, consists in taking the definition of truth to be relative to two time coordinates, and similarly to two world coordinates. Taking S, as before, to be a structure of interpretation, and f an assignment of values to the free variables, here is what we get:

\[
\begin{align*}
[[\text{FUT}\varphi]]^S_{w_1,w_2,t_1,t_2,f} & = T \\
\text{iff for some } t \text{ later than } t_1: \quad [[\varphi]]^S_{w_1,w_2,t,t_2,f} & = T
\end{align*}
\]
In other words, there are two time parameters, $t_1$ and $t_2$; $t_1$ is the time of evaluation, on which, as before, non-indexical tense operators such as FUT operate, while the other time parameter, $t_2$, is deployed in the definition of indexical tense operators. What ‘now’ does is to reset the value for $t_1$ by assigning to it the current value of $t_2$. Similarly, there are two world parameters, $w_1$ and $w_2$; $w_1$ is the world of evaluation, on which, as before, non-indexical modal operators such as $\Box$ operate, while the other world parameter, $w_2$, supplies the value for $w_1$ whenever the interpretation comes upon the modal indexical ‘actually’.

With the indexical ‘now’ and the above definitions at our disposal, we can account for that third, problematic reading of (7) by giving it the following formal rendering:

9  $\text{FUT} \forall x (\text{NOW} (\text{flourishing}(x)) \rightarrow \text{faded}(x))$

It is not difficult to show that the truth conditions associated with (9) in virtue of the proposed definitions of FUT and NOW capture the truth conditions intuitively associated with (7) on the reading under consideration. For, suppose that (7) is uttered at a certain time $t$, and consider what needs to be the case for (9) to come out true when evaluated at $t$. There needs to be a time later than $t$ at which everything that was flourishing at $t$ is faded. Thus, for instance, in the scenario described above, if (9) is evaluated at the time at which the begonias and the hortensias are flourishing, but after which there is no time when they are (simultaneously) faded, (9) will come out false.

3  The Logic of Indexicals

We have seen, in the last section, that context-dependence per se is not inconsistent with a truth-conditional, model-theoretic approach to natural language. It must be noted nevertheless that the kind of context-dependence
that words such as ‘now’, ‘today’ or ‘actually’ manifest is special, in that context-dependence is built into the word’s meaning itself. The meaning of the word ‘now’ thus tells you that the time referred to by a given occurrence of this word is the time of the context in which the word is being used. Similarly, ‘today’ means that the day it picks out is the day of the context. Words whose context-dependence is constitutive of their very meaning are called indexicals and include, beside indexical temporal and modal adverbs that we came across in the previous section, demonstrative and personal pronouns such as ‘this’, ‘I’, ‘you’, etc.\(^6\) Indexicality is, then, that sort of context-dependence which is lexically encoded, hence constitutive of the very meaning of a given expression. If, in addition, we think of logical truth as truth in virtue of meaning,\(^7\) the question arises of what are the logical truths involving indexicals, and whether there are any interesting such truths that are not already obtainable from standard logic (such as “If it’s raining now, then it’s raining now”, which is arguably just an instance of the logically valid schema \(\varphi \rightarrow \varphi\)).

David Kaplan famously held that there is a non-standard logic of indexicals. (Indeed, his interest in logic was one of the driving motivations for his work on the semantics of indexicals.) Consider the following:

10 I am here now. (uttered by Kaplan on April 23, 1973 in Los Angeles).

Kaplan noted that (10) is “deeply, and somehow universally true” (unlike the sentence “Kaplan is in Los Angeles on April 23, 1973”), because “one need only understand [its] meaning to know that it cannot be uttered falsely” (1977: 509), and concluded that (10) ought to come out logically true. The logical framework that he devised incorporates double indexing, but in addition to the world and the time parameter that are used in interpreting the modal indexical ‘actually’ and the temporal indexical ‘now’, he

\(^6\)Indexicality is a more narrow phenomenon than context-dependence, which is arguably ubiquitous and not always formally tractable. Some, like Travis (1996), have even held that the phenomenon of context-dependence poses a major threat to the very enterprise of truth-conditional semantics. The view that context-dependence cannot be reconciled with a truth-conditional, model-theoretic approach to meaning (which, in a way, was Frege’s own view) is nowadays referred to as ‘radical contextualism’, and gave rise to some debate in recent years. For discussion, see e.g. Recanati 2004, Predelli 2005, Preyer and Peter (eds.) 2005, to mention only a few sources representing various sides of the debates.

\(^7\)It is possible to consider truth in virtue of meaning as analytic truth, without necessarily thinking that all analytic truths are logical truths or vice versa. For the sake of simplicity, I shall not draw such a distinction between analytic and logical truth, to avoid the risk of being taken astray.
introduced an agent parameter, to be used in interpreting the first person
pronoun ‘I’, as well as a location parameter for ‘here’. The sequence of
those four parameters is what constitutes, in Kaplan’s framework, the pa-
rameter of context, with the proviso that only sequences (a, p, t, w) such
that the agent a is located at the location p at time t and in world w may
qualify as contexts. The full parametrization of truth, in this framework,
deploys a structure, a world and a time of evaluation, an assignment of val-
dues to free variables, and a context. From there, Kaplan defined a derived
notion of truth in a context as follows:

11. sentence \( \varphi \) is true in structure S, with respect to assignment f, in
context c iff \( [[\varphi]]_{c,w(t(c)),f}^S = T \), where \( w(c) \) is the world of c and
t(\( c \)) is the time of c.

Given the definition in (11), and the above proviso on contexts, what is
peculiar about the sentence in (10) is that it comes out true in every context,
which is what makes it, as Kaplan would say, “somehow universally true”,
and endows it with the status of a logically valid sentence.

The putative logical validity of “I am here now” is one of the notorious
examples, but also one of the most controversial. The reason why Kaplan
thought that that sentence was logically valid is that he thought that its truth
was entirely warranted by the lexical meaning of the sentence. But he was
wrong in this respect. The truth of (10) is warranted by the lexical meaning
 plus the additional assumption that the speaker must be located at the place
of utterance (=here) at the time of utterance (=now). That this is indeed an
additional assumption is clear from the fact that Kaplan only admits among
the contexts of his model-theoretic structures those quadruples (a, p, t, w)
that satisfy the proviso that a is located at p in w at t, i.e. such that the
pair (a, p) belongs to the value that the interpretation function of a given
structure yields when applied to the pair (t, w). But if such a stipulation is
required in order to make (10) come out logically valid, then the presumed
logical validity of this sentence is not very convincing to begin with. For, it
is clearly not a matter of logic, but an empirical matter, to determine who
is located where at what time. Yet, for Kaplan, this matter must be settled
before one can even decide whether something is an admissible structure
of interpretation for the logic of indexicals. \(^8\)

\(^8\) For further discussion of this problem, see Stojanovic (2008a): 41-43 and Predelli and
Stojanovic (2008). Let me add that the idea that “I am here now” is logically true has also
been challenged on the grounds that there may be true utterances of the negation of that
sentence, such as a message on an answering machine that says “I am not here now”.

Let us therefore look at another, somewhat less controversial example. Consider:

12 If it is actually raining now in Mumbai, then it is raining in Mumbai.

To most ears, (12) will sound logically true (or valid). But the validity of (12), if validity there is, raises a problem. For, it is also plausible to consider the following two rules to be truth-preserving, logically valid rules:

- **the necessitation rule**: if $\varphi$ is valid, then $\Box \varphi$ is also valid
- **the eternalization rule**: if $\varphi$ is valid, then $\text{ALWAYS} \varphi$ is valid

Applied to (12), the rules will give us that the following sentence is also a truth of logic:

13 Necessarily, it is always the case that if it’s actually raining in Mumbai now, then it’s raining in Mumbai.

But this is, as Kaplan pointed out, an unwelcome consequence. Suppose that it is actually raining in Mumbai now. Then it follows from (13) that it’s always raining in Mumbai in all possible worlds. But it isn’t always raining in Mumbai. Hence (13) cannot be true.

One of the achievements of Kaplan’s work, and of the double-indexing strategy more generally, is to provide a way of making sentences such as in (12), and, more generally, instances of the schema (14) below, logically true, without *ipso facto* making (13) and, more generally, instances of (15) below, true:

14 **ACTUALLY NOW** ($p \rightarrow p$)

15 $\Box \text{ALWAYS} \text{(ACTUALLY NOW} (p \rightarrow p))$

Assume a semantics that deploys two world parameters and two time parameters, as in the framework outlined in section 2, and consider the following definition of validity (or logical truth):

**Def. 1.** Sentence $\varphi$ is valid if, and only if, for every structure $S$, every assignment function $f$, and for every world $w$ and every time $t$, we have:

$[[\varphi]]_{w,w,t,t,f}^S = T$
This definition, in a way, collapses the two parameters when it comes to defining logical truth. To see this, contrast Def. 1 with the following alternative definition:

**Def. 2.** Sentence $\phi$ is valid if, and only if, for every structure $S$, every assignment function $f$, and for all worlds $w_1, w_2$ and all times $t_1, t_2$, we have: 
$$[[\phi]]^S_{w_1, w_2, t_1, t_2, f} = T$$

Note that on neither definition does (15) come out valid. However, only on Def. 1 are sentences such as (14) valid. On Def. 2, (14) is not valid precisely because the truth of ‘actually now (p)’ will depend on $w_2$ and $t_2$, while the truth of $p$ alone depends on $w_1$ and $t_1$. And since there is no correlation between $w_1$ and $w_2$, or between $t_1$ and $t_2$, the antecedent of the entailment in (14) may be true while the consequent is false.

How should one adjudicate between the two definitions? Is it even correct to assume that there is one correct definition of validity in this case? For it may be that there are two equally valid notions of validity, so to speak: the one that corresponds to truth in every context, and the other that corresponds to truth for all assignments of values to all the parameters to which truth is relative, and that allows that different worlds serve as the world of evaluation and the world of the context; and similarly for times. The primary aim of this paper was not to answer these questions, but rather, to raise them and revive them as open issues for further investigation that lie at the interface of semantics, logic and philosophy of language.

**References**


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9 This “collapsing” of the two parameters is sometimes referred to as “diagonalization”, echoing the work of Robert Stalnaker (1998).

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