Topological Essentialism

Roberto Casati, Achille Varzi

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Roberto Casati

CNRS, Séminaire d'Epistémologie Comparative, Aix-en-Provence (France)

Achille C. Varzi

Department of Philosophy, Columbia University, New York, New York (USA)

Abstract. This paper analyses topological variants of mereological essentialism, the thesis that an object cannot have different parts than the ones it has. In particular, we examine *de dicto* and *de re* versions of two theses: (i) that an object cannot change its external connections (e.g., adjacent objects cannot be separated), and (ii) that an object cannot change its topological genus (e.g., a doughnut cannot turn into a sphere). Stronger forms of structural essentialism, such as morphological essentialism (an object cannot change shape) and locative essentialism (an object cannot change position) are also examined

1. INTRODUCTION

Your left and right hands are now touching each other. This could have been otherwise; but could your hands not be attached to the rest of your body? Sue is now putting the doughnut on the coffe table. She could have left it in the box; but could she have left only the hole in the box? Could her doughnut be holeless? Could it have two holes instead? Could the doughnut have a different hole than the one it has?

Some spatial facts seem tainted by necessity. This is problematic, since spatial facts are a paradigm of contingency. But the intermingling of space and modality may be surprisingly intricate. To a degree this is already visible in the part-whole structure of extended bodies. Parthood, itself a *prima facie* extrinsic relation, has an uncertain modal status. And questions about the necessity or the contingency of spatial facts and relations seem to run parallel to questions about the necessity or the contingency of parthood relations.

Consider: Could an object have different parts than the ones it has? Common sense has an easy, affirmative answer to this question. However,

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there are philosophers who, pressed by the need to overcome difficult co-nundrums concerning the identity of spatio-temporal particulars, have cast doubts on the adequacy of the common-sense answer. In recent years, for instance, Roderick Chisholm [1973, 1975, 1976] has defended the radical view that a true individual can neither gain nor lose parts, so that each single part is essential to it—a view that has come to be known as *mereological essentialism.*

> Let us picture to ourselves a very simple table, improvised from a stump and a board. Now one might have constructed a very similar table by using the same stump and a different board, or by using the same board and a different stump. But the only way of constructing precisely *that* table is to use that particular stump and that particular board. It would seem, therefore, that that particular table is *necessarily* made up of that particular stump and that particular board. [1973: 582-583]

This is by no means an uncontroversial view.[1] It does, however, have some philosophical appeal and it is not in itself utterly implausible. *Strictly speaking*, if the parts change, so does the whole.

Now, mereology accounts for one basic way of conceiving of an object's spatial structure, its decomposition into parts. But this basic account can be extended in various ways by further distinguishing how the parts can be related to the whole (and to one another). Some parts can lie in the interior of the whole; others can reach out to the surface. Some parts can overlap; others can be connected to one another without sharing any parts. Even the notion of a whole that we get by reasoning exclusively in terms of parthood can be significantly enhanced by distinguishing between wholes that are all in one piece, such as a stone or a cup, and scattered wholes made up of several disconnected parts, such as a broken glass, a set of suitcases, an archipelago. These distinctions run afoul of mereology and call for concepts and principles that are distinctively topological. Topology, one might say, is a natural complement of mereology. It supplies a notion of wholeness to complement the theory of parthood; it supplies a theory of connection besides the theory of overlap.

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1. See e.g. Plantinga [1975], Wiggins [1979], Van Cleve [1986], Willard [1994].

Call *mereotopology* the result of integrating these two theories: the theory obtained by supplementing the basic mereological apparatus with sufficiently articulated topological distinctions. There is a sense in which the difference between mereotopology and mereology is only a matter of descriptive or expressive power: mereotopology is more fine-grained than mereology. But this is hardly a remarkable metaphysical difference. Where the two theories start to behave differently is in their metaphysical consequences. And this is where the intermingling of space and modality may reveal unexpected oddities. Regardless of how one feels about mereological essentialism, if mereology generalizes to mereotopology then it is natural to inquire about the possible topological extensions of essentialism. We may, for example, speak of topological essentialism to indicate the strengthening of Chisholm's doctrine that results from insisting on the essentiality of an object's topology (over and above its mereology). Is the table necessarily in one piece? Would we have another table were the stump detached from the board (that very stump and that very board)? Is Sue's doughnut necessarily torus-shaped? Would it be something else had its hole shrank out of existence (e.g., because the doughnut was squeezed into an amorphous blob)?

The proxies of mereological essentialism in the mereotopological domain seem to have interesting and yet unexplored consequences. As in the case of mereological essentialism, some extrinsic relations turn out to be intrinsic relations. (The stump's being attached to the board becomes an essential property of the stump—and of the board.) But exactly what that means and exactly what relations are at issue is unclear. What follows is a first step in the assessment of these matters.

2. MEREOLOGICAL AND TOPOLOGICAL ESSENTIALISM

Let us begin by defining the basic notions. Mereological essentialism is not a clear-cut thesis, and various formulations have been considered. To represent some of these formulations in a uniform way, and also to lay out some convenient notation for comparisons with other forms of essentialism considered below, it will be advantageous to have some general way of expressing the

modal statement that a certain proposition holds in every world in which a given entity exists. For this purpose, where φ is any formula, we shall rely on the following general notation:

$$(1) \qquad \Box_x \varphi =_{df} \Box(E!x \to \varphi).$$

Here the symbol '$\Box$' in the definiens is to be understood as the modal operator for necessity and 'E!' as the predicate of singular existence, definable e.g. as in free logic:

$$(2) \qquad E!x =_{df} \exists y\, y=x.$$

So (1) is simply a compact way for expressing the modal statement that φ holds in every world in which x exists. (We may refer to '$\Box_x$' as the relativization of $\Box$ to x.)[2]

Using this notation, we can immediately express the thesis of mereological essentialism in terms of parthood ('P') as follows:

$$(PE) \qquad Pxy \to \Box_y\, Pxy.$$

That is, if an entity x is part of an entity y, then x is part of y in every world in which y exists.[3] Note that this can take on different meanings depending on the properties of 'P'. Typically this is axiomatized as a reflexive, antisymmetric, transitive relation (a partial ordering), but there is some disagreement as to what additional properties are characteristic of the parthood relation. Particularly in modal contexts, the behavior of 'P' is far from clear, and this can be crucial in assessing the full import of (PE). For instance, let 'O' stand for the relation of overlap, defined as usual as sharing of a common part:

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2. This is a modal statement whose intended interpretation presupposes some sort of possible world semantics. In the spirit of greater generality, one could also consider formulations of the thesis that are compatible with a counterpart-theoretic understanding of modality.

3. Some authors would add the requirement that x must also exist in those worlds in which y exists and x is part of y. We shall not consider this part of the story and refer the reader to Simons [1987], Chapter 7, for discussion. Likewise, we shall ignore here the variants of mereological essentialism that can be obtained by making parthood interact with time or with other parameters, as initially suggested by Plantinga [1975].

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$$(3) \qquad \mathrm{O}xy =_{\mathrm{df}} \exists z(\mathrm{P}zx \wedge \mathrm{P}zy),$$

Then the analogue of (PE) for 'O' yields a natural, alternative formulation of the thesis of mereological essentialism:

$$(\mathrm{OE}) \qquad \mathrm{O}xy \to \square y\, \mathrm{O}xy.$$

However, it is easy to see that the two formulations are not exactly equivalent unless parthood is assumed to satisfy the so-called principle of the remainder:[4]

$$(\mathrm{RP}) \qquad \mathrm{PP}xy \to \exists z(\mathrm{P}zy \wedge \neg\, \mathrm{O}zx),$$

where 'PP' indicates proper parthood:

$$(4) \qquad \mathrm{PP}xy =_{\mathrm{df}} \mathrm{P}xy \wedge x \neq y.$$

More precisely, (PE) logically implies (OE), but in the absence of (RP) the converse implication may fail. A counterexample is schematically illustrated in Figure 1, which features the Hasse diagrams of two mutually accessible worlds each consisting of three objects a, b, and c (parthood relations go uphill along the lines). These worlds violate the remainder principle (RP), since in both cases every part of b overlaps a even though a is a proper part of b. As a result, the model defined by w and w' falsifies (PE), since a is part of b in w but not in w'. Yet (OE) holds.
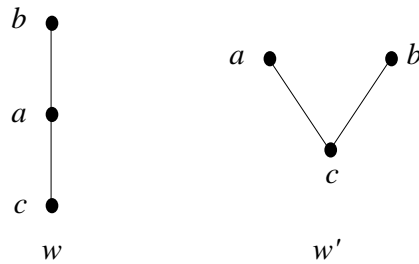

Figure 1: A model satisfying (OE) but not (PE)

4. This principle is called "Weak Supplementation Principle" by Simons [1987: 29], who considers it a minimal requirement for any partial ordering to deserve the name of a part. Simons reserves the label "Remainder Principle" for a stronger form asserting the existence of a unique difference between an entity and any proper part thereof.

These differences are not without interest, but should not concern us too much. It does not matter which of these versions one takes to be representative of the view traditionally associated with the label "mereological essentialism". (Chisholm himself follows (PE).) Nor is it our purpose here to assess the viability of these principles. Rather, let us see how these purely mereological statements can be extended to mereotopological statements.

To this end, let us use 'C' as a primitive expressing the topological relation of connection. This is a reflexive and symmetrical relation, like O; however, on the intended interpretation C is more general than O because things can be connected even in the absence of sharing of parts.[5] Accordingly, one can define various distinctively topological notions such as external connection (EC), tangential part (TP), boundary part (BP), or internal part (IP):

$$(5) \quad \mathrm{EC}xy =_{df} \mathrm{C}xy \;\wedge\; \neg\, \mathrm{O}xy$$
$$(6) \quad \mathrm{TP}xy =_{df} \mathrm{P}xy \;\wedge\; \exists z(\mathrm{EC}zx \;\wedge\; \mathrm{EC}zy)$$
$$(7) \quad \mathrm{BP}xy =_{df} \exists z(\mathrm{P}zx \;\wedge\; \mathrm{TP}zy)$$
$$(8) \quad \mathrm{IP}xy =_{df} \mathrm{P}xy \;\wedge\; \neg\, \mathrm{TP}xy$$

We can then immediately consider two versions of topological essentialism, corresponding to (PE) and (OE) respectively. The first can be formulated in terms of the relation of interior parthood, IP (tangential parthood would do as well):

$$(\mathrm{IPE}) \quad \mathrm{IP}xy \;\rightarrow\; \Box y\, \mathrm{IP}xy.$$

The other can be formulated in terms of the connection relation C:

$$(\mathrm{CE}) \quad \mathrm{C}xy \;\rightarrow\; \Box y\, \mathrm{C}xy.$$

Also in this case, the two formulations are not equivalent. In fact neither implies the other unless 'C' is constrained by additional axioms. More interestingly, note that although C includes O and P includes IP, there are no analogous inclusion relations between the corresponding essentialist principles. Concerning C and O, consider first two externally connected objects

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5. The exact characterization of 'C' is itself a matter of controversy. See Varzi [1996] and Smith [1996] for an examination of the issues involved.

that are, in another world, disconnected: (OE) is vacuously satisfied, but (CE) fails (case (i) in Figure 2). On the other hand, consider two overlapping objects that are, in another world, externally connected. (CE) is satisfied but (OE) is falsified (case (ii)). Concerning IP and P, consider first an object which, in some world, loses some of its boundary parts: (IPE) is vacuously satisfied but (PE) fails (case (iii)). On the other hand, consider an object whose internal parts become, in another world, tangential parts: (PE) is satisfied but (IPE) is falsified (case (iv)).

w   w   w   w

w'   w'   w'   w'
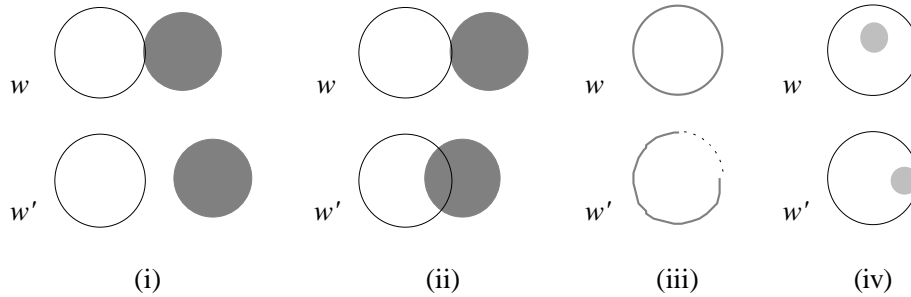
(i)   (ii)   (iii)   (iv)

Figure 2: Relative independence of mereotopological essentialist principles

There are also stronger (conceptually, if not logically) forms of topological essentialism that one might consider. For instance, one could hold that external connection is a binding relation, in the sense that if two entities are externally connected, then they are necessarily so:

(ECE)   ECxy $\Box y$ ECxy.

Likewise for all the other interesting mereotopological relations that can be defined in terms of 'P' and 'C', which we need not consider here.

At this point the extended picture of essentialism begins to look non-trivial. There are innocent cases of extension of mereological essentialism to topological essentialism. For instance, the necessity of overlap just is the necessity of (non-external) connection. (If your left hand necessarily overlaps your body, then your left hand is necessarily connected to your body.) That is simply a matter of definitions. The qualifying case is that of external connection (EC). This is logically included in connection, like overlap, hence the

necessity of external connection extends to the necessity of connection. But surely one would like to say that two objects that are actually attached could lie apart from each other. One would like to say that the hand *could* be detached from the rest of the body. Thus, even if one accepted mereological essentialism, one would seem to have obvious reasons for rejecting at least some of its topological extensions.

3. TOPOLOGICAL ESSENTIALISM REVISITED

Well, are these reasons so obvious? What notion do we have of things that are connected? Prototypical examples are constituted by a thing and its geometric complement (the entity that results when we imagine the thing as having been subtracted from the universe as a whole); or by the interior of an object and its boundary; or perhaps by a hole and its material host (like the hole in Sue's doughnut). In the temporal realm, typical examples are two successive events such as a process and the event starting with the process's culmination. In general, if connection is understood in the spirit of ordinary topology, then external connection is a relation that can only hold between two entities one of which is closed by a boundary (in the contact area) and the other is "open" (in the same area). In all of these cases the thesis of topological essentialism does not sound unreasonable. *If* it is plausible to say that the parts of an object are essential to it, then it is likewise plausible to say that its complement is essential to it, or that its interior is essential to its boundary (and so on). We are not saying that this is the correct view: it is hardly controversial on a *de dicto* reading, since on the intended interpretation of these terms nothing can lie between an entity *x* and *the rest* of the universe (at least, this is true as long as the connection relation is assumed to satisfy certain plausible conditions); but on a *de re* reading there is ample room for controversy. Even so, the idea that the connection between an entity and its complement (or between a boundary and its interior) has the modal force of necessity is not in itself utterly implausible—not any more implausible than mereological essentialism.

The cases in which the principle(s) of topological essentialism do sound implausible arise when the things whose contact is said to be necessary are

ordinary objects, such as a book and the shelf it stands on, or a painting and the wall it hangs on. There is indeed a lot of *prima facie* implausiblility in the statement that these things are necessarily touching—that the book could not be on a different shelf, or that the painting could not be hanging on a different wall. There is a lot of implausibility in the thesis that the book or the painting would literally cease to exist or no longer be what they are upon removing them from the shelf or the wall with which they are presently in contact. But on closer inspection, these are not counterexamples to topological essentialism, in any of its forms. They are not counterexamples because the relevant relation of contact here is not one of topological connection. The book and the shelf, or the painting and the wall, are not externally connected. They are very close; but closeness is no mark for connection. (The book and the shelf, or the painting and the wall, do not make up connected units, things in one piece.)

To be sure, natural language does not distinguish between these two senses of "connection", between true topological connection and mere physical closeness. In general, the surfaces of distinct physical objects cannot be topologically connected, though the objects may of course be so close to each other that they appear to be truly connected to the naked eye. They are, as we may say, quasi-connected (QC). And this is not a case covered by (IPE) or (CE). We can, of course, express a correspondingly strong essentialist principle to the effect that any two things that are quasi-connected are so in all worlds in which they exist:

(QCE)   QCxy $\Box y$ QCxy.

But this is no innocent topological extension of mereological essentialism. It is a much stronger and independent, substantive thesis—a form of *metric* essentialism whose rebuttal would not affect the question of the plausibility of topological essentialism, and which therefore does not have much bearing on the question of the relationship between mereology and topology. Our initial example of your two hands touching also falls into this case. In any event, even in this regard one could find some plausibility in the essentialist thesis, at least relative to certain kinds of entity. Would that mosaic be the same if the tesserae were arranged differently? Would that Tinkertoy house be the same if the Tinkertoys were arranged in totally different way?

Would this beautiful bunch of flowers be the same if the flowers were scattered all over the floor?

There is, finally, the case of contact between two parts of a given object, such as your hand and the rest of your body, or two halves of a solid sphere. They are connected. Are they necessarily so? Here it seems that the intuitive plausibility of an affirmative answer would indeed mark a much stronger commitment than mereological essentialism. It seems quite plausible to maintain that the two halves are essential to the sphere without entertaining the additional view that they cannot be separated—that the sphere cannot be cut in half. We agree with this. But, once again, we would like to suggest that things look different on closer inspection. Surely the two parts can be detached (no essentialist thesis would deny that the sphere is *divisible*). The question is whether they would survive the separation—whether the connected halves are the same things as the disconnected halves. Equivalently, the question is whether the sphere survives the cut—whether the connected sphere and the split sphere are one and the same thing. It would seem natural to answer these questions in the affirmative. After all, *nothing* happened to the two halves, except for a change in their extrinsic, relational properties. And how could a mere "Cambridge change" result in a substantial change?[6] However, this is hasty if our concern is with topological facts, for topological relations need not be of the Cambridge variety. The splitting transforms the two halves into full-fledged, complete, maximally connected things. Two entities that were only partly bounded by a surface now are completely bounded and perfectly separated from their complement. Or, if you prefer, two merely potential entities have become actual. On either view, the change is remarkably substantial: the parts remain the same, but the boundaries change. And to say that nothing happened, to deny that the two halves undergo a dramatic change, is to beg the question. If a mereologist can be struck by microscopic mereological changes, a mereotopologist has all the reasons to be struck by such macroscopic topological changes.

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6. A recent formulation of this sort of thinking can be found in Denkel [1995] *contra* Burke [1994]. See also van Inwagen [1981]. The intuition can already be found in Philo of Alexandria's criticism of the Stoic puzzle of Theon and Deon: see Sedley [1982].

Nor should one be puzzled by the role played by surfaces in this account. There is a natural worry that reference to surfaces may give rise to unsolvable philosophical conundrums: what happens to the surface when we cut an object in half? Which of the two halves is going to inherit the property of being topologically closed? This is indeed a problem if we think that dissecting a solid reveals matter in its interior and "brings to light new surfaces", as Adams [1984: 400] nicely put it. But this is not the right intuition here. Rather, the model we should have in mind, if we wish to understand what happens topologically when the sphere is cut in half, is that of a splitting oil drop: eventually the right and left portions split, and we have two drops, *each with its own complete boundary*. (Think also of a soap bubble splitting.) There is nothing mysterious in this process, except for the fact that an abrupt topological change (much more dramatic than a mere Cambridge change) takes place.[7]

With all this, topological essentialism appears to be stronger but in no way stranger than its pure mereological counterpart. The last case is the only one where one would need additional grounds for arguing from mereological essentialism to the thesis that the two halves of a sphere cannot be separated. But even so, this brief discussion shows that such additional grounds are a rather natural extension of whatever grounds one could offer in favor of mereological essentialism.

4. WHEN MEREOLOGY FORCES TOPOLOGY

The link between mereological and topological essentialism becomes especially vivid once we realize that some topological facts are already made necessary by the assumption of mereological essentialism. This at least is true if mereology is assumed to be strong enough to satisfy the principle of the remainder (RP).

Consider internal parthood (IP). If x is in the interior of y in the actual world w, then y cannot become an interior part of x. This follows directly from mereological essentialism via the transitivity of parthood and the princi-

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7. This point is argued in detail in Varzi [1997].

ple of the remainder. For if *x* is in the interior of *y*, then by (RP) there must exist some part *z* of *y* which does not overlap *x*. Now suppose there is a world *w'* in which *y* is contained in *x*. In this world, *z* would have to be a proper part of *x* by transitivity; but then, it should be necessarily so by mereological essentialism.[8] Thus, in particular, *z* would have to be part of *x* in the initial world *w*, contrary to the assumption. Hence if *x* is contained in the interior of *y* in *w*, *x* cannot contain *y* in any world accessible from *w*. And this is a matter of *de re* necessity.

Another interesting case is the necessary asymmetry of tangential parthood (TP). If *x* is a tangential part of *y*, then *y* cannot be a tangential part of *x*. (Proof along the same lines.) The reason for the irreversibilty of internal and tangential parthood lies, of course, in the asymmetry of proper parthood. Joined with the remainder principle and mereological essentialism, this freezes the *nesting* of parts. Hence, in particular, it freezes the asymmetries corresponding to interior and tangential proper parthood. On the other hand, the asymmetry of proper parthood does not freeze the *relative position* of parts, and that is why such principles as (IPE) of (CE) do not follow from mereological essentialism: even in the presence of the remainder principle, the formal properties of proper parthood are not enough to ensure the *de re* necessity of *interior* parthood or, more generally, to fix the *de re* necessity of the parts' relative positions. (See again case (iv) in Figure 2.) These are independent topological necessities.

For a third example, consider again the relation of connection between an entity and its mereological complement. As we have seen, the *de dicto* necessity of this relation is trivial: whatever the modal variations of *this* object or *this* complement, nothing can ever lie between an object and the rest of the universe. The *de re* variant of the object/complement necessary connection is more controversial. But it, too, turns out to be a consequence of mereological essentialism as long as certain minimal assumptions are made concerning the logical structure of parthood. It is enough to assume the remainder principle (RP) along with the—rather innocent—assumption of the existence of the universe (the mereological sum of all there is in a world). For

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8. Here we are assuming the relation of accessibility among worlds to be symmetric.

consider a world *w* in which *y* is the complement of *x*. If in another world *w'* it turns out that *y* is no longer in contact with *x*, then there must be a part of the universe, *z*, which belongs neither to *x* nor to *y*. By (PE), *z* must be a part of the universe in *w* too. Therefore it must overlap either *x*, or *y*, or both. But if it does, it is a contingent fact, as shown by *w'*, and this is ruled out by (OE).

5. SPHERES AND DOUGHNUTS

There are other senses in which topological essentialism may not seem unreasonable, for there are other forms of topological essentialism besides those expressed by modal statements such as (CE), (IPE), (ECE), and the like. These statements express a clear sense in which the essentialist position can be extended from mereology to mereotopology by insisting on the essentiality of such characteristic relations as interior parthood, tangential parthood, etc. However, one can also think of the essentiality of topological properties of a different sort, such as the properties as being topologically sphere-like, doughnut-like, and so on—in short, those properties that determine the topological *genus* of an object. Correspondingly, one can formulate principles of topological essentialism that force the genus of an object to be the same in every possible world. This is the sort of essentialism that arises in relation to our initial questions about Sue's doughnut.

There are, again, a *de dicto* and a *de re* formulation of such principles. Let us write 'D*x*' for '*x* is a doughnut' and 'H*yx*' for '*y* is a hole in *x*'. Then we could express the *de dicto* formulation as follows:

(D)   □ *x*(D*x*   *y* H*yx*)

This seems uncontroversial (if not analytically valid). Surely you cannot have a holeless doughnut, for having a hole is part of being a doughnut: in every world in which you have a doughnut, you have a hole in it (just as in every world and for every husband there is a corresponding wife.) There is nothing peculiar about topology here, and there are perfectly comparable forms of *de dicto* mereological necessity: surely you cannot have a sphere with the right half removed. A truncated sphere is just not a sphere.

The interesting questions arise with the *de re* formulation. Given a

doughnut, is it true of it—of *that* particular object—that it could not exist without its hole—*that* very particular hole? To be sure we can specify this claim in at least two different ways:

$$\text{(DE1)} \quad Dx \quad \Box x \, \exists y \, Hyx$$
$$\text{(DE2)} \quad Dx \quad \exists y \, \Box x \, Hyx$$

The first of these is a weaker claim, as it says nothing about whether a given doughnut could have a different hole from the one it actually has. The second is a much stronger claim and rules out the possibility that a doughnut could be perforated by a different hole: in every world in which you can find Sue's doughnut, you will also find the very same hole that it has in this world. Both claims of course have various generalizations, concerning for instance the number of holes in an object and eventually also its topology (a doughnut with a knotted hole). Here we may content ourselves with the simple cases.

Is either of these theses acceptable? On one intuition they are both too strong. If you open up a doughnut (e.g., by cutting it on one side) the hole goes, the doughnut stays. That is, the *object* is still there, and you can still eat it all, though its shape is now different. This would contrast both (DE1) and (DE2). On the other hand, there is no a priori reason why this commonsensical intuition should be regarded as a *reductio ad absurdum* of either (DE1) or (DE2). Concerning (DE1), there is actually a close connection between this and the topological essentialist theses expressed by (CE) and (ECE). If you open up the doughnut, some parts that were connected become disconnected. So if (CE) or (ECE) hold, (DE1) must hold too. (However the converse need not be true, hence (DE1) is effectively less committing than the other theses.)
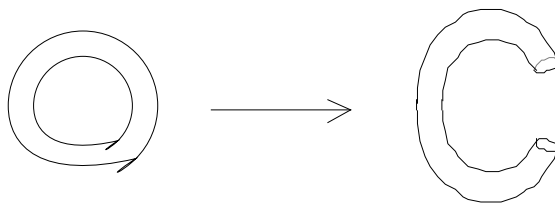

Figure 3. Opening up a doughnut kills the hole in it. Does the doughnut survive?

Moreover, there is also a connection between (DE1) and the mereological essentialist theses expressed by (PE) and (OE). The hole in a doughnut is part of the doughnut's complement, and a tangential part at that. Thus, to the extent that the relation of external connection between every object and its complement is a *de re* necessity that follows from mereological essentialism (as seen in the previous section), to that extent the relation of external connection between a doughnut and its hole is itself a matter of *de re* necessity supported by mereological essentialism. So if (PE) and (OE) hold, (DE1) must hold too.

These considerations do not extend to (DE2). Yet also in that case reference to common sense and intuitions is hardly a way to settle the issue. If the thesis that the each part of the doughnut is essential to it can survive the cry of common sense, so may the thesis that the hole—that very hole—is also essential.[9]

Of course, each of (D), (DE1), and (DE2) was phrased in terms of a binary predicate, 'H', which reflects a reifying attitude towards holes: holes are entities of a kind and can stand in various relations with the other specimens of the ontological fauna. This is a view that we have defended elsewhere.[10] However we can also imagine an expression of the form '$yHyx$' to be a metalinguistic abbreviation for a complex expression attributing to x the property of being perforated, "without any implication that perforation is due to the presence of occult, immaterial entities" (as Argle once put it [11]). On this account, the possession of a hole by a doughnut would simply be a *façon de parler*, and "there are holes in ..." would be an innocuous shape predicate like "... is a doughnut". This has no consequences for the *de dicto*

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9. If one thinks of holes as negative parts (as suggested e.g. by Hofmann and Richards [1985]), then the entire story here would reduce to plain mereological essentialism. That hole would be necessarily in that doughnut insofar as the hole is a part of the doughnut—a negative part. Rather than an advantage, however, we take this to be an element against such an account of holes: there is an important distinction here—a distinction reflected in the possibility of endorsing topological essentialism without endorsing mereological essentialism, or vice versa. And this distinction is obscured by a negative-mereological treatment of holes.
10. See Casati and Varzi [1994].
11. See Lewis and Lewis [1970].

principle (D), which in fact becomes synonymous with the statement that necessarily a doughnut is a doughnut. However the status of the *de re* principles changes on this account. For one thing, the strong formulation (DE2) becomes meaningless (syntactically ill-formed). As for (DE1), the thesis remains intelligible, but it becomes an instance of a more general thesis to the effect that an object must necessarily have the shape it has. If sameness of shape is understood topologically (i.e., modulo topological equivalence), then the result is equivalent to (DE2), and bears the same relations to the other essentialist principles of mereology and topology that were noted above. If, by contrast, sameness of shape is taken literally, then the result is a much stringer thesis, which may be labeled *morphological* essentialism. Can an object have a different shape than the one it has? Does it cease to exist when its shape changes? Whether this is metaphysically acceptable, or *when* it is acceptable if at all, is a new question—another step into the intricate intermingling of space and modality.

6. LOCATIVE ESSENTIALISM

We can go even further in that direction. Consider strengthening the link between an object and the region of space at which it is located by asserting the necessity of location. This yields a form of *locative* essentialism which is stronger than any form of spatial essentialism considered so far. To be more precise, let 'L' indicate the relation between an object *x* and "its" region of space, r*x*: Basic features of this relation are injectivity (if L*xy* and L*xz* then *y*=*z*, so that location is, in this sense, *exact* location) and conditional reflexivity (regions are located at themselves).[12] Then the thesis of locative essentialism can be stated thus:

$$(LE) \qquad L xy \quad \Box x\, L xy.$$

That is, an object that happens to be located at a certain region is necessarily located there, whenever it exists.

Note that (LE) implies that no object can survive with impunity a rear-

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12. For a more detailed treatment of this relation, see Casati and Varzi [1996].

rangement of its parts. This is stronger than mereological essentialism (PE) and topological essentialism (CE), since a rearrangement of parts need not carry a loss of parts—violating (PE)—or a change in the topology of the object—violating (CE). For instance, in figure 4, one can observe a displacement of parts with no mereological or topological change. This would falsify (LE) while being compatible with both (PE) and (CE).
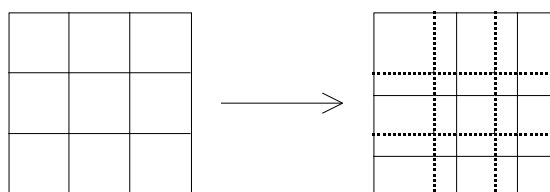

Figure 4. (PE)–(OE) and (CE)–(IPE) are preserved; (LE) fails.

Now, like its mereotopological analogues, principle (LE) expresses *de re* modalities. There are, to be sure, weaker *de dicto* versions of locative essentialism, which can be formulated in terms of the region operator r. For instance, the following gives a *de dicto* counterpart of (LE),

(L)   □ x y (rx=y Lxy),

and this is obvious. Surely x is located at y whenever y qualifies as the region at which x is located. This is the same sort of triviality that we encountered in the *de dicto* versions of the other principles. The corresponding *de re* forms are certainly questionable principles. Presumably the painting that is now on that wall could have also been on this wall. And presumably the book that is now on the shelf could very well have been on the coffee table. To deny this seems to be an intolerably severe position—certainly much more severe than the position expressed by the above principles of topological essentialism, let alone mereological essentialism. As a matter of fact we are not aware of any philosophers who ever held such views. But consider, for an analogy, van Inwagen's words about the temporal case:

> It is bad enough to suppose that the replacement of a rusty bolt leaves me with what is, "in the strict philosophical sense", a new car. It is infinitely

worse, and never had the phrase 'infinitely worse' been used more appropriately, to suppose that when I sit in my car and turn the wheel, what I am occupying is, "in the strict philosophical sense", a compact series of infinitesimally differing cars. [1990: 77-78].

This statement can be rephrased to match our concerns. It is very bad to suppose that, had the wheel of my car been three degrees more to the left than it actually is, the car would have been another entity.

Similar considerations apply, it would seem, to various other forms of locative essentialism that could now be formulated, thanks to the many distinctions afforded by a mereotopological vocabulary. For instance, one could consider weakenings of (LE) to the effect that if the region occupied by an object x stands in a certain relation with the region occupied by another object y, then it must necessarily stand in that relation. The generating scheme is

$$(\text{LE}) \qquad \text{r}xry \quad \Box x \text{ r}xry.$$

where is any mereotopological predicate such as 'P', 'IP', 'O', 'C', and the like. Statements of this form are generally weaker than (LE) (except when y is a region and is the identity relation, which gives (LE) as a special case). Yet van Inwagen's misgivings would still seem to apply: it is infinitely hard to imagine that the car you are sitting in would be another thing had its region not been, say, an interior part of the region occupied by the garage.

Is there any use for these forms of essentialism, then? Note the difference between (LE) and the forms of topological or metric essentialism exemplified by (CE) and (QCE). Those are principles concerning the relative positions of certain objects: (CE) demands that two halves of a sphere be necessarily in contact with each other, but leaves room for the possibility that they jointly move around in the environment; (QCE) demands that the book be necessarily on the shelf, or that the tesserae of the mosaic be necessarily arranged as they are, but leaves room for the ideal possibility that the book be moved along with the shelf, or that the mosaic be somewhere else (insofar as this does not collide with other essentialist facts). By contrast, (LE) concerns the relative position of spatial regions. Now, on the *de dicto* reading,

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this is much more exorbitant than any of the above: (LE) becomes a thesis to the effect that things cannot be elsewhere—whether individually or together with other entities in the environment, a thesis beside which even the strongest forms of mereological and topological essentialism pale. However, on a *de re* reading (LE) is commensurable to the corresponding forms of topological or metric essentialism. In fact, insofar as spatial regions are entities of a kind, (LE) says neither more nor less that those entities are necessarily -related, which is what (CE) and the like say. (A relationalist about space could hold analogue views, though the actual content of the principles would of course be different. Specifically, one can obtain relationist analogues of the principles of locative essentialism by replacing each occurrence of a binary predication 'L*xy*' with the idiom 'L**x*' that gives its relationist translation.)

On a *de re* reading, then, (LE) is not an independent thesis. Just as mereology forces certain topological facts, so topology forces certain locative facts—those facts that concern the location of spatial regions. This yields an asymmetry between (LE) and (LE), since (LE) was seen to be independent of both mereological and topological essentialism. It does not follow, however, that (LE) is unintelligible. Consider then a purely spatial model—a model in which every world is inhabited exlusively by regions of space (including Space, the fusion of all these regions). Assume both mereological and topological essentialism, in any of the forms discussed above. Then the corresponding instances of (LE) are true. But then (LE) must also be true. For this principle states that regions cannot be located elsewhere. So how can it be violated, if the only interpretaton of this 'elsewhere' is given in term of localization itself?
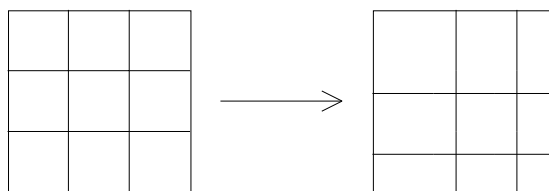

Figure 5. In a spatial domain, (LE) supervenes on (PE)–(OE) and (CE)–(IPE).

There is—we may say—a holistic character in the decomposition of space into regions. In a spatial domain, the difference between the two chessboards in figure 5 is merely decorative, for regions cannot be anywhere else than where *they themselves* are. In a spatial domain, the distinction between *de re* and *de dicto* reading of (LE) collapses, and the entire essentialist story supervenes on matters of mereology and topology.

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