True and False: An Exchange

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TRUE AND FALSE: AN EXCHANGE

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On a sunny day, on the seashore, Tactic and Tictac receive their first message in a bottle. They are good at radical interpretation. They master logic pretty well too. And they have independent evidence that ' ' and ' ' are sentential connectives (for "disjunction" and "conjunction", as they have learned to say).

"Look, Tactic says, look at this—a disjunction!" He holds it up:

$$p \quad d$$

"Either p or d. I wonder who wrote that, and why they sent it to us in a bottle. How exciting!"

Tictac grabs the paper from his hands. "Wait a moment . . . that is not a disjunction. It is a *con*junction. Don't you see?

$$p \quad d$$

Both p *and* d—that's what it says. Not a disjunction—a conjunction. No doubt about that."

"Come on, Tictac, don't mess things up. Obviously you are reading the page upside down. If you turn a disjunction around it may well look like a conjunction—that's a peculiar thing about those connectives, and about Pees and Dees. . ." Ponders. "Oh, dear—maybe you are right. That *could* be a conjunction. *And* it could be a disjunction."

"So what does the message say, then?", asks Tictac.

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"I'm afraid we can't tell. We know it is either a disjunction or a con-junction—but that's about it. They forgot to indicate the orientation!"

"I see . . ."

"On the other hand—Tactic continues—suppose we maximize truth (a charitable move). Say the message expresses a true proposition. Then we can ask: *How* could it be true? And if we find out what 'p' and 'd' stand for, then perhaps we can solve the puzzle."

Tictac shakes her head: "No way. I just realized that we made an un-warranted assumption, and a wrong one at that. This is not the disjunction or the conjunction of two different propositional letters, for if one of them is part of this language the other cannot be also in the language—remember? Negation is expressed simply by writing the negated sentence upside down.[1] (That's how you get the principle of double negation: flip the sentence twice, and you'll get it back.) So if 'p' is part of the language (and let's pretend we have independent evidence for that), 'd' cannot be a *Dee*, otherwise the language would be ambiguous. Hence it must be an inverted *Pee*, it must ex-press the negation of p."

Tactic looks puzzled. "I'm not sure I'm following you. Even if 'd' ex-pressed a negation, what I just said should remain valid. Actually, if that *is* the negation of p, and if the message expresses a true proposition, then you must admit I had it right in the first place. That can only be a disjunction—a tautology, indeed. Surely not a contradiction!"

"Well, suppose the message comes from some paraconsistent logician, or some philosopher who actually believes some contradictions are true. Then what are we to make of your charitable assumption? In that case I could be right—the sentence could be a conjunction."

"I see . . ."

"Besides, even if we stick to classical logic (as I concede we should, if we want to make some progress), that could still be the sign for conjunction. That sentence might very well be *the negation of a contradiction*. Take my p-and-not-p (I mean, p   d), and turn it upside down: a tautology, but no occurrence of the disjunction sign ' '."

"But this means that the language is ambiguous after all. If negation works that way, then the very same expression could stand for, say, a

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[1] Evidently, Tactic and Tictac took Ramsey's suggestion seriously: 'Facts and Propositions', *Proceedings of the Aristotelian Society*, Suppl. Vol. 7 (1927), 153–170.

conjunction, '*A* and *B*', and also for a negated disjunction, 'not-(not-*B* or not-*A*)'."

"Yes. This is classical logic, though. If you buy it, you must keep it. Just like *A* and not-not-*A* are logically indistinguishable, so are the expressions you just mentioned. But the ambiguity is always *salva veritate*; i.e., the two sentences are always logically equivalent. Hence the ambiguity is negligible."

"So, is our dispute a mere consequence of the peculiar way negation is expressed in the language?"

"Not quite. I think we could have the same problem even without this collapse of logical equivalence and identity. Suppose one expressed negation by encircling the negated sentence rather than by turning it upside down. Then a conjunction '*A* and *B*' would be perfectly distinguishable from the equivalent negated disjunction, 'not-(not-*B* or not-*A*)', and a disjunction '*A* or *B*' would be perfectly distinguishable from the equivalent negated conjunction, 'not-(not-*B* and not-*A*)'. Yet we might still be in a position of indetermination."

"How?"

"Just pretend our language had the peculiarity of including sentence symbols that share with ' ' and ' ' the fortuitous property of being reversible along the horizontal axis, like the letters 'p', 'd', 'q', 'b'. (No syntactic ambiguity in this case, since negation is explicitly marked.) Then some sentential expressions would remain well-formed upon reversing them. And it would be impossible to tell a disjunction from a conjunction unless we knew *how to read it* in the first place. Consider this:

$$\text{p} \quad \text{ⓟ}$$

A tautologous disjunction? A contradictory conjunction?"

"So you agree—that can only be a conjunction sign if the whole expression were a contradiction."

"Yes, Tactic—in that case I would have to assume the message comes from a paraconsistent logician, or else give up your charitable assumption that the sentence is meant to express a truth."

"So back to our message, then. The point is, unless we rule out that it expresses a negation or a contradiction, we can't figure out which connective is being used (a token of ' '? a token of ' '?) because we have no way of telling *from the message alone.*"

"Right."

"Well, meaning *is* determined by inferential role, or so we have learned. So perhaps we could settle our dispute if we only knew what can be validly inferred from that sentence, or from what the sentence could be validly inferred."

Tactic has just finished speaking and a second bottle arrives. This time it is Tictac who opens it. Out comes a little machine.

"Well, says she, it looks like an inferential mechanism, it's set up like an inferential mechanism—blamed if I don't believe it *is* an inferential mechanism."

"And a classic one it is, says Tactic, enthusiastic. Let's feed our message into it!"

Tictac puts the paper with the message into the opening, turns the handle, and an instant later the machine expels a card with a new message on it.

$$p$$

"See? Then it *was* a conjunction—says Tictac. A conjunct always follows from a conjunction, we know that. It's called -elimination in the book. (Even most paraconsistent logicians would agree, I should think.) So I was right after all. I knew I was right."

"Now you wait a moment, Tictac. What *I* see here is that a disjunct follows from a disjunction."

"But that's not a classically valid pattern of inference, Tactic! It is not even paraconsistently valid."

"Well, let's see. Is this really supposed to be a truth-preserving inferential machine?"

"I guess it is, yeah."

"Well, then why would they send us a contradiction in the first place? *Everything* follows from a contradiction. Or if you prefer, there is no *truth* to be preserved if you start with a contradiction."

"OK, then, I take it back. Maybe this is not a machine to derive truths from truths, but a machine to derive falsehoods from falsehoods. Maybe the sentence we got was a contradiction, and now we have a machine that takes care of the logic of contradictions—the logic of falsehoods, I should say. How about that?"

"Fine with me. But then you must admit that I was right. If the machine implements a calculus to prove contradictions, we have just witnessed an application of the relevant rule of ∧-elimination (not ∨-elimination): if p ∧ d is a falsehood, then so is p. Let's see what happens if we put p back in …"

The machine answers:

$$p \lor q$$

"What did I tell you? This is the logic of falsehoods: every conjunction involving a falsehood is itself false."

"Do I need to point out that, to me, this now reads as a disjunction?

$$b \lor d$$

You asked what follows from d (that is, not-p), and the machine's answer was that if that is true, then so is any disjunction having it as a disjunct. No surprise about that!"

"But then I'm even more confused than I was at the beginning. If this is an ordinary machine for truth-preserving inferences, why did you say the initial message expressed a contradiction?"

"I didn't say it expressed a contradiction. I said the connective was a conjunction. Maybe it was a contradiction; maybe it was a tautology. Remember: turn a contradiction around, and it becomes a tautology."

Little help came from a third attempt, recognized at once by both as inferentially valid:

$$\text{inputs: } p \lor q, d$$
$$\text{output: } q$$

"If we are dealing with truth, then this was a simple disjunctive syllogism. But I'm afraid we can also take it the other way around,

$$\text{inputs: } b \land d, p$$
$$\text{output: } b$$

and that gives us a valid pattern for the calculus of falsehoods. Let me try once more . . ."

"No, Tactic, there's no point in trying again. We know what's going to happen. Tautologies and contradictions—truth and falsehood—are so per-

fectly dual that we cannot even *say* whether this is going to be a logic of truth or a logic of falsehood. Truth is reversed falsehood, and falsehood is reversed truth. The duality is perfect." [2]

"But in general duality is compatible with the existence of relevant differences. We can, for instance, agree to write '–1' for '1' and '1' for '–1' and still use the minus sign for the function taking a number to its opposite partner. Yet the difference in notation would show up in the truth of such equations as $1 \cdot 1 = -1$ or $-1 \cdot -1 = -1$." [3]

"I agree. But our case is worse. It is not only a question of duality. In our case, the principle of double negation sneaks into the picture and weeps out all the differences. And the machine can't help us, because that principle holds in both logics—the truth-logic and the falsehood-logic."

"What a disappointment, Tictac . . . I was so excited about the message in the bottle. I really wanted to find out. "

"I'm disappointed too. Still, we have learned our lesson for today, have we not? If negation is classic, we may always be fooled—we may always be seeing logical constants through a mirror. Logical constants are determined only up to duality. Their meaning may well be determined by their role in a logical calculus; but even assuming we know that the calculus implements a fully classical logic (and even assuming we know the syntax in full—as we in fact do), there is no way to tell whether the implementation goes via truth-preserving principles or falsehood-preserving principles. How do we know whether these folks have sent us an inferential mechanism of the first kind rather than one of the second kind? Both would do the same good job, after all. Both would characterize classical logic equally well."

"Right. We are used to viewing logic as a tool for truth-maintenance. ('Is that true? Then good, this is true as well.') But perhaps these folks view logic as a tool for falsehood-avoidance instead. They might be suspicious people—obsessed by falsehoods and concerned first and foremost with avoiding them. ('Is that false? Then beware, this must also be false.') That is

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[2] C. G. Morgan, 'Sentential Calculus for Logical Falsehoods', *Notre Dame Journal of Formal Logic*, 14 (1973), pp. 347–353.

[3] The example is from P. M. Simons, 'Frege and Wittgenstein, Truth and Negation', in R. Haller and J. Brandl (eds.), *Wittgenstein – Towards a Re-Evaluation. Proceedings of the 14th International Wittgenstein Symposium*, Vol. 1, Vienna: Hölder-Pichler-Tempsky, 1990, pp. 119–129.

how they might approach logic.[4] And that is how this machine they sent us might work."

"That is exactly the point, my dear Tactic. We tend to posit assent, but these people might expect us to posit dissent."

"And to us this may seem a gratuitous change of orientation.[5] But how can we ever be sure? How can we know whether the theorems of their calculus are meant to be tautologies or contradictions?

"Yes. They have to tell us, somehow."

"Let us say they have to tell us how we are supposed to hold those messages."[6]