A pragmatic approach to the problem of logical omniscience

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Abstract

I distinguish two perspectives on the analysis of belief/knowledge sentences, which I call *metarepresentational* and *representational* respectively. I argue that they need not necessarily agree and use the distinction to clarify the so-called problem of logical omniscience. The acceptance of the regularity rule $\phi \leftrightarrow \psi \therefore K\phi \leftrightarrow K\psi$, in particular, doesn't amount to a principle of closure of knowledge under logical equivalence provided it is interpreted as a metarepresentational principle. I attempt to show that its apparently undesirable consequences are in fact pragmatically filtered out in the context of ordinary ascriptions. The principle of distribution of knowledge over conjunction is discussed along the way.

'It is one question what the objects of belief are, another question how the 'that'-clause of a belief sentence specifies the object of the ascribed belief'

D. Lewis ([10]: 411)

1 Introduction

The definition of what should count as an adequate logic of knowledge is highly problematic. To most people, a logic of knowledge is nothing but a particular interpretation of one of several possible systems of modal logic. Such a view is easily illustrated, but says nothing indeed of the adequacy requirements that constrain the choice of one system rather than another. One would like an adequate system of modal epistemic logic to give 'acceptable' axioms and rules of inference, but acceptable with respect to what? Is it primarily with respect to a specific epistemological theory of the agents' capacity to process the content of their knowledge, or rather with respect to the way ordinary belief or knowledge ascriptions are made in conversational contexts? I call the first perspective on the content of knowledge *representational*, and the second *metarepresentational*. My aim in this paper is to show that these two perspectives on the content of knowledge are sufficiently independent in principle to motivate axiomatic principles that might be conflicting. I argue that this distinction is often neglected however, and that some long-standing problems in the logic of knowledge, such as the so-called problem of logical omniscience, are in fact ambiguous.

I will focus on the evaluation we make of inferences of the form:[1]

(1) X knows that ϕ
(2) $\phi \equiv_i \psi$
(3) X knows that ψ

where X stands for an agent, ϕ and ψ for statements, and '$\phi \equiv_i \psi$' means that the sentences ϕ and ψ are supposed to be equivalent for specific notion of semantic equivalence, where the index variable $i \in I$ is meant to make clear what notion of semantic equivalence between statements is considered relevant. Thus, '$\phi \equiv_i \psi$', depending on the value of i, can mean that the two statements only have the same truth-value; that the two statements are logically equivalent; that the two statements are analytically equivalent; that the one results from the other by an appropriate substitution of coreferential singular terms, or coextensional predicates, *etc.* For instance, Quine's paradigmatic example (4)-(6):

(4) Philip knows that Tully denounced Catiline
(5) Tully denounced Catiline $\equiv$ Cicero denounced Catiline
(6) Philip knows that Cicero denounced Catiline

is an example of inference where the embedded statements are equivalent in having the same truth value and resulting from each other by substitution of the coreferential proper names 'Cicero' and 'Catiline'.

Without further syntactic and semantic analysis, the range of the notion of semantic equivalence between statements remains largely open and problematic. A systematic description of the data, hence of the structure of the index set I can be made more precise, but I will leave it aside here.[2] One way to set things tight is indeed to oversimplify the underlying syntax of the language, as well as the relevant notion of equivalence. Thus, my purpose will be

[1] Throughout this paper, I shall speak indifferently of 'knowledge sentences' or 'belief sentences'. I take 'epistemic sentences' to mean both. Nothing here depends on the distinction between knowledge and belief, and the reader is invited to make the substitution wherever he or she finds it more appropriate.

[2] I refer to [5] for a detailed analysis of the data. In [5] I distinguish five notions of semantic equivalence: *coextensionality, necessary equivalence, analytic equivalence, logical equivalence, and synonymy.* The pattern of inference (1)-(3) is more accurately described in the following way, to which I shall more readily resort when appropriate:

X knows that $\phi[t]$
$t \equiv t'$
X knows that $\phi[t']$

where t is a syntactic component of the sentence ϕ; t and t' have to be of the same syntactic category; and '$t \equiv t'$', expresses that those are semantically equivalent for an appropriate equivalence relation. The metavariable t can stand for a singular term, a definite description, a predicate, or a sentence. Thus t can be the sentence ϕ itself.

chiefly to clarify the problem of the substitution of logically equivalent statements under the scope of the knowledge/belief operator in the framework of a system of monomodal propositional epistemic logic. I think, however, that one cannot have an adequate grasp of this problem unless one also takes into account the inferences of substitution involving singular terms and predicates.

This paper is divided into two main parts. In the first part, I start out discussing Recanati's recent theory of metarepresentations and its application to belief reports, and offer to flesh out the distinction between a representational and a metarepresentational approach of epistemic sentences by contrasting Hintikka's and Montague's original analyses. In the second part, I propose to clarify the so-called problem of logical omniscience: I argue that the usual formulation of the problem has both a representational and a metarepresentational side, and that the metarepresentational one can be given a pragmatic solution.

2 Metarepresentational and representational accounts of epistemic sentences

2.1 Recanati's theory of metarepresentations and opacity

Statements of the form 'X knows that ϕ' are statements of knowledge attribution. When uttered in a specific context, they express the representation of a given speaker on the state of knowledge of an agent X. To the latter corresponds a specific representation as well. Recanati ([14]) therefore considers knowledge attributions as only a subclass of a wider class of statements that he calls *metarepresentational* statements. Metarepresentational statements express representations about representations. The class of metarepresentational statements also includes metafictional statements, such as: 'In the film, Sailor loves Lula'. The sentence: 'Sailor loves Lula' expresses a specific object-representation, whereas the prefix 'in the film' makes clear that this representation is seen from the point of view of an outside spectator.

One way to rephrase the above problem is to ask: which inferences of substitution are legitimate within the context of metarepresentational epistemic operators? Recanati discusses two types of examples, and gives his solution to the problem. Consider the following inferences involving susbtitution of synonymous predicates and coreferential proper names respectively:[3]

(7) Pierre knows that John is an eye-doctor
(8) an eye-doctor $\equiv$ an ophtalmologist
(9) Pierre knows that John is an ophtalmologist

(10) Pierre knows that Hesperus is a planet
(11) Hesperus $\equiv$ Phosphorus
(12) Pierre knows that Phosphorus is a planet

Both instances exemplify the pattern described above: the embedded statements correspond to each other by substitution of synonymous predi-

[3] Henceforth, I use the pattern described in footnote 2.

cates in the first case, and from substitution of coreferential proper names in the second. Are these inferences valid? Recanati ([14]: 41) considers that a general principle of intensional replacement applies in both cases: two genuine singular terms having the same referential content, like two predicates expressing the same concept, can be substituted to each other in any complex sentence. This principle also applies to two sentences expressing the same proposition. The predicates 'eye-doctor' and 'ophtalmologist' are intensionally equivalent in this sense, as well as the proper names 'Hesperus' and 'Phosphorus'.

Recanati does not specify the sense in which this notion of intensional equivalence is to be cashed out, although the possible world framework can serve to define it: both predicates or proper names will have the same extension in every possible world (and in the case of proper names, this extension will be the same *across* possible worlds, by the rigidity thesis). Likewise, two sentences will express the same proposition if and only if they have the same truth-value at every possible-world. Short of explaining whether concepts, individuals, or propositions are to be individuated in precisely this way however, Recanati's analysis may prove insufficient, in particular because it doesn't allow to see what this general principle of intensional replacement really amounts to, and to what extent it is discriminating. In particular, a fine-grained theory of intensions could make this principle straightforward, while it may appear counter-intuitive in the case of traditional possible-worlds semantics.

We shall come back to this objection later. For now, the important and interesting thing lies mainly in the way Recanati offers to justify the principle. Thus he writes ([15]: 395):

> According to Hintikka (1962: 138-141), failures of substitutivity in belief contexts show that two co-referential singular terms, though they pick out the same individual in the actual world, may refer to different objects in the ascribee's belief world. That option is ruled out in the present framework; for we want the ontology to remain that of the ascriber all along: we want the singular terms to refer to the same objects, whether we are talking about the actual world or about the ascribee's belief world. That is the price to pay for semantic innocence.

This commitment to the sole ontology of the ascriber is a strong methodological constraint. The idea is that the substitution of cointensional expressions under epistemic prefixes need not imply a change in the so-called mode of presentation of the content ascribed to the agent. Of course, many contexts of attribution are contexts in which the inferences above are problematic, for instance if Pierre is asked 'did you know that John was an ophtalmologist?', and Pierre answers 'no' where he would have assented to the question 'did you know that John was an eye-doctor?'. For that reason, Recanati concludes that attitudinatives are expressions *dependent* on the ascriber's communicative intention ([15]: 392). Failures of substitutivity are accountable in terms of the presence of a so-called unarticulated constituent in the prefix[4], of the form:

(13) Pierre so-knows that John is an eye-doctor

where the modified prefix 'so-knows' is meant to point out the contribution of the very *form* of the sentence 'John is an eye-doctor' to the truth conditions of the attribution, or more generally, the contribution of a specific mode of presentation of which the sentence then serves as an *index*, in the sense of Nunberg ([12]).

Recanati's solution is illustrated for failures of substitution of either synonymous predicates, or corefential singular terms. Recanati does not discuss examples of substitution of entire sentences however. This is certainly a weakness of his analysis once again, since he doesn't say whether the principle of intensional replacement to which he subscribes goes along with a classical, montagovian theory of propositions, or with a more fine-grained theory of propositions. In other words, Recanati doesn't say whether any two true mathematical sentences, for example, express the same proposition, as is notorious in the case of possible-world semantics. If that is the case, then, the common reluctance to accept an inference of the form[5]:

(14) Pierre knows that 2+2=4
(15) Pierre knows that $sh(x) + ch(x) = e^x$

will also have to be explained by the presence of an unarticulated constituent in the epistemic prefix. The inference should then be read as:

(16) Pierre so-knows that 2+2=4
(17) Pierre so-knows that $sh(x) + ch(x) = e^x$

Such a solution would in fact be reasonably close to the the one successively entertained by R. Montague himself, and later by R. Stalnaker for whom mathematical knowledge is individuated by the mathematical sentences, and not solely by the (always true) proposition these sentences express.[6] Yet, Recanati's solution is different in one important respect: it should imply that, for

[4]The question whether it is appropriate to account for this modification of the prefix by the presence of an unarticulated constituent is not relevant to the problem under examination, and I do not discuss it here.

[5]'$sh(x)$' is the expression of the hyperbolic sinus function, and '$ch(x)$' of the hyperbolic cosinus function. The relation '$sh(x) + ch(x) = e^x$' holds of any real number x.

[6]See Montague ([11]), and Stalnaker ([16]), *passim.* For instance, Montague writes ([11]: 137): 'A second objection might concern the fact that if ϕ and ψ are any logically equivalent sentences, then the sentence

$$\mathcal{B}[J, \hat{}\phi] \rightarrow \mathcal{B}[J, \hat{}\psi]$$

is logically true, thouth it might under some circumstances appear unreasonable. One might reply that the consequence in question seems unavoidable if propositions are indeed taken as the objects of belief (...) and that its counterintuitive character can perhaps be the ascriber, the two-sentences '2+2=4' and '$sh(x) + ch(x) = e^x$' do express the same proposition. But this is certainly doubtful, if only because in the case of mathematical statements, the ascriber also should make an obvious distinction of meaning between any two syntactically distinct mathematical statements. For that reason, the idea of systematically reading 'knows' as 'so-knows' in the case of mathematical statements is not natural, and it would be more reasonable to give a theory of propositions capable of individuating mathematical propositions more finely from the start.

Despite the absence of discussion of the problem of substitutivity at the level of whole sentences, Recanati's theory remains consistent and appealing, insofar as:

(a) it distinguishes very sharply between the point of view of the ascribee and that of the ascriber in the case of a knowledge report.

(b) up to intensional equivalence, it leaves room for an entire freedom of substitution in the way the content of her attitude is ascribed to an agent. Furthermore, it suggests that the level of object-representation of the ascribee certainly involves specific modes of presentation, although these modes of presentation are not to be encoded in the semantic content of the embedded clause.

(c) it therefore clearly suggests that when an attitudinal content is ascribed to an agent up to intensional equivalence, that does not entail that the agent herself should be able to express the content of her attitude under as many guises as there are syntactically distinct ways of ascribing this content. Consequently, it draws a clear line between the semantic principle of intensional replacement, and the pragmatic restrictions that might be imposed on it in some given contexts.

I will try to show that this defense of the semantic principle of substitutivity can be adapted to the more specific case of epistemic logic, although my argument will not primarily resort to a theory of *dependent* epistemic operators. Before that, I would like to flesh out the distinction between a representational and a metarepresentational approach of knowledge sentences.

2.2 Representational *versus* metarepresentational approaches to attitude ascriptions

Even though one may disagree with Recanati's treatment of attitude reports, and despite the gaps which we pointed out in his analysis, we have seen one strong motivation of his theory, namely the sharp distinction made between what the ascriber endorses when she makes a knowledge or belief attribution, and what the ascribee is meant to endorse. In other words, an inference such as :

(16) Philip believes that Cicero denounced Catiline
(17) Cicero $\equiv$ Tully

traced to the existence of another notion of belief, of which the objects are sentences or, in some cases, complexes consisting in part of open formulas'.

(18) Philip believes that Tully denounced Catiline

can remain entirely ascriber-oriented, and does not require, to be valid, that Pierre know the invididual x named 'Cicero' and 'Tully' under both guises.

More generally, the difference emphasized by Recanati between his treatment of opacity and Hintikka's treatment can be generalized in the following way: I call *representational* a semantic treatment of knowledge ascriptions that purports to describe the content of knowledge or belief from the point of view of the ascribee's epistemic architecture, and *metarepresentational* a semantic treatment of knowledge ascriptions that focuses on the description that an ascriber is prone to make of the putative content of knowledge this agent. The distinction is methodological in the first place. A rough way to put it might be to say that a representational approach of knowledge/belief sentences is thought-oriented, while a metarepresentational analysis of the same sentences is language-oriented. But the latter terminology is clearly too vague and less informative than the 'representational-metarepresentational' one, which points to the duality of agent and reporter.

This methodological distinction has important semantic consequences, as witness the differences between Hintikka's truth-conditional analysis of belief sentences as expressed in (19), and Montague's analysis as expressed in (20):

(19) 'X knows that ϕ' is true in w iff ϕ is true in all epistemic alternative worlds accessible from w to agent X

(20) 'X knows that ϕ' is true in w iff X is suitably related to the proposition expressed by ϕ

I think there are two main differences between both analyses, that are too often neglected in discussions surrounding epistemic logic. These differences are relative to : a) the scope and function of possible worlds b) the axiomatic principles of knowledge induced by each semantic analysis respectively. I focus here on the first point, and will consider the second in my discussion of the problem of logical omniscience.

In Hintikka's analysis, the set of possible worlds is essentially relative to the ascribee, and more generally, the analysis of knowledge statements is not supposed to be relative to their utterance by a speaker in a given context. Hintikka focuses on the way an agent can reach a state of knowledge by gradual elimination of uncertainty (in the form of scenarios compatible with a given information state). In Montague's intensional analysis, the set of worlds is not ascribee-relative, and the reason for that is that Montague's motivation was linguistic prior to be psychological. More precisely, the worlds are supposed to be only the worlds relative to the universe of discourse of the ascriber; they are not intended to represent the possibilities that are relative to the information state of a given agent. Although Montague sometimes underlined the psychological plausibility of his analysis, he was interested in the inferences a given ascriber is likely to make in the first place, and not so much in the way a knowledge state may be defined relative to the ascribee.[7] Even though both analyses are cashed out in terms of possible worlds, the space of possible worlds turns out to be different in each case.

This difference is significant in the Tully/Cicero case, for example. Using the framework of quantified modal logic to represent the knowledge of agent Philip, the inference from (16)-(17) to (18) may be blocked simply by supposing that the two proper names do not pick out the same individual in all of Philip's belief worlds. As Maria Aloni notes in her evaluation of this hintikkean solution to the problem of substitutivity:

> The failure of substitutivity of co-referential terms (in particular proper names) in belief contexts does not depend on the ways in which terms actually refer to objects (so this thesis is not in opposition with Kripke (1972)'s analysis of proper names), it is simply due to the possibility that two terms that actually refer to one and the same individual are not believed by someone to do so. ([1]:44-45).

By contrast, in the framework of Montague grammar, where the worlds are not relative to the ascribee, the adoption of a theory of rigid designation for proper names validates the inference (16)-(18), because in this case, the two proper names refer to the same individual in all worlds. We have seen from the above quotation of Recanati that his commitment to the principle of semantic innocence, according to which the semantic value of an expression remains the same inside as well as outside of attitude contexts, also validates the inference (16)-(18) in principle. This common feature of Montague's and Recanati's analysis is well accounted for by the fact that the 'ontology' underlying the semantic principle is meant to be 'the ontology of the ascriber all along'. This feature is characteristic, once again, of what I call a metarepresentational approach.

3 Logical omniscience

3.1 Logical omniscience in epistemic logic and intensional logic

I now turn to the second difference mentioned above, which deals with the axiomatic epistemic principles induced by or compatible with a specific semantic analysis of belief sentences. Hintikka's original logic of knowledge and belief was philosophically concerned with the principles governing the behavior of epistemic operators. These principles are supposed to be characteristic of the epistemology of knowledge and belief. For instance, a principle like

[7] See Montague (1970), p. 138-139, and the quotation in footnote 6 above. The fact that Montague took the attitude of believing itself to be a relation between agent and propositions, and not only the denotation of the 'that-clause' in the context of a belief ascription to be a proposition, suggests that Montague's approach was not only metarepresentationally but also representationally driven. Recanati, on the contrary, never conflates the two perspectives, he talks only of the objects of belief as they are ascribed, not as they really are.

positive instrospection ($K\phi \rightarrow KK\phi$, or (KK) for brief), should be adopted only on account of a specific theory of belief or knowledge *qua* attitudes. In his original defense of the principle (KK), Hintikka intimates that his arguments for the principle are 'logical rather than quasi-psychological'([8]: 111). This remark is supposed to rule out one specific argument from introspection: it remains that Hintikka clearly distinguishes between the normative dimension of what he calls a 'virtual equivalence' between $K\phi$ and $KK\phi$, and the restrictions which may affect the principle in the context of ordinary ascriptions of knowledge.[8] Likewise, in his recent attack of the principle, Williamson clearly rests his argument on epistemological (as well as metaphysical) considerations (Williamson [18], c.4). Montague's analysis of belief sentences, on the contrary, was meant to be entirely neutral with respect to the principles governing the behavior of knowledge or belief as such.

The theories of knowledge ascriptions of Montague and Hintikka are similar in resorting to the possible world framework, although the space of possible worlds is not the same from the one to the other. But they are also similar in sharing the notorious problem of licensing the substitution of any two necessarily true (or necessarily false) statements under the scope of a knowledge verb. This problem is an aspect of the more general 'problem of logical omniscience'. However, and it is worth repeating, Montague's semantic analysis of belief sentences was not philosophically concerned with the principles governing the transition of one state of knowledge or belief to another. The fact that epistemic logic and intensional logic licence the same rule of inference tends to convince many people that if this rule of inference is a problem for one framework, it is thereby a problem for the other, and conversely that if one is able to give a solution for one framework, one will thereby be able to transpose it to the other one. This parallelism is misleading however: it is not because both analyses use the possible world framework and thereby share a common rule of inference that this very rule should be given the same status in both. To see the point, it is worth stating the problem of 'logical omniscience' in its general form.

The problem of logical omniscience is a problem pertaining to the closure properties of belief or knowledge vis à vis the object-representations these attitudes govern. The problem can be described schematically thus: *if agent X knows that ϕ, and the sentence ϕ bears a specific metalogical relation R to ψ, does it follow that X know that ψ?* For instance, the relation R may be defined as: $\phi R \psi$ iff $\phi = (\psi \wedge \gamma)$ or $\phi = (\gamma \wedge \psi)$. In this case the problem is whether the knowledge of agent X is closed under conjunction. The question of closure at stake here is clearly more general than the problem of substitutivity discussed so far, in which we assumed the embedded statements to be semantically *equivalent*, ie to have the same meaning (for some relevant notion of semantic equivalence).

First I claim that the problem of logical omniscience in its most general

[8] See [8]: 112 *sqq.*

form is a representational problem. It is an epistemological and logical problem prior to being a problem about ascriptions. The question is whether it is legitimate to build a theory of knowledge (resp. belief) in which knowledge (resp. belief) obeys such and such closure principle. The problem is no longer to determine which linguistic variations are admissible in the *description* of the content of knowledge of an agent; it is to determine the range of logical operations by which a given epistemic content may be transformed systematically into a distinct, logically related content.[9] This is the sense of questions like: is knowledge closed under material implication, under conjunction, or under logical consequence? The usual Hintikka-Kripke semantics for epistemic logic implies that the knowledge of a given agent encompasses all the tautologies of the propositional calculus, is closed under conjunction, under material implication, as well as logical consequence. These various forms of closure are rightly considered irrealistic when referred to the knowledge of a human agent, or considered as normative idealizations.

Secondly, the so-called problem of logical omniscience arising in Montague's intensional logic is more specific than the problem characteristic of any normal modal logic for belief or knowledge. In intensional logic, the theory of propositions predicts that belief contexts licence the substitution of logically equivalent expressions. This closure phenomenon is weaker than closure under logical consequence. Schematically, the following principle is valid in intensional logic, where j is an individual constant, ϕ and ψ are sentences, and $\mathcal{K}$ is a knowledge or belief predicate of the appropriate type:

$$\text{from } \models \phi \leftrightarrow \psi, \text{ it follows that} \models \mathcal{K}[j, \hat{}\phi] \leftrightarrow \mathcal{K}[j, \hat{}\psi]$$

This principle has a natural axiomatic modal propositional logic counterpart, known as the rule of regularity, where K now is a modal sentence operator:

$$\text{from } \vdash \phi \leftrightarrow \psi, \text{ it follows that } \vdash K\phi \leftrightarrow K\psi \tag{RE}$$

Besides, this principle is the only modal rule of inference that one needs to add to an axiomatization CP of the classical propositional calculus where modal formulas are allowed to appear in the axiom schemata, in order to get a sound a complete axiomatization of the so-called Montague-Scott semantics for modal logic (a.k.a. *neighbourhood semantics*) presented by Fagin & alii ([6]: 316-321). I will call **MS** this axiomatic modal theory, consisting of CP, and the two rules of inference (MP) and (RE).

Montague-Scott semantics for propositional epistemic logic is in fact the exact modal counterpart of Montague semantics for the full intensional theory of types, in that it captures the informal analysis given in (20) above. It should

[9] It needn't be assumed that the objects of belief or knowledge *qua* attitudes are sentences in order to make sense of the problem of the closure properties of knowledge as it is stated above. What is needed is only the assumption that to the distinct metalogical relations holding between belief sentences, there correspond well-defined logical relations between the epistemic contents these sentences refer to.

be thought of as a 'small' intensional logic. An **MS**-model is a modified Kripke model $\mathcal{M} = \langle W, C, V \rangle$, where W is a non-empty set of worlds, V is a usual valuation function associating a set of worlds (or *proposition*) to each atom, and C is a function associating to each world a set of subsets of W. If w is a world, $C(w)$ yields a set of sets of worlds, ie a set of *propositions*. These propositions are taken to be the propositions known by the agent at world w. The rule of evaluation corresponding to (20) now becomes: $\mathcal{M}, w \Vdash K\phi$ iff $\{s \in W; \mathcal{M}, s \Vdash \phi\} \in C(w)$.

A noteworthy feature of this semantics is that the set of worlds can now be assumed to be ascribee-independent, as explained in the previous discussion of Montague's full intensional logic: one need not define a specific relation of epistemic accessibility between worlds.

In their presentation of Montague-Scott semantics, Fagin & alii write :

> All forms of logical omniscience fail except for closure under logical equivalence. In other words, while agents need not know all logical consequences of their knowledge, *they are unable to distinguish between logically equivalent formulas*. This is as much as we can expect to accomplish in a purely semantic model, since logically equivalent sentences are by definition semantically indistinguishable ([6]:319, italics mine).

This interpretation of Montague-Scott semantics clearly suggests that the substitution of logically equivalent sentences is ascribee-oriented. I think this interpretation is misguided however: one way to contest that the substitution of equivalent sentences is ascribee-oriented is to consider to the contrary that the principle of substitution of logically equivalent sentences is a metarepresentational principle, and not a representational principle. I call here *metarepresentational* a principle that is likely to govern our belief or knowledge attributions, and *representational* a principle that is supposed to govern the way the objects or attitudes of knowledge and belief are likely to be connected. The principle of positive introspection (KK) is clearly a representational principle in this sense.

An open question is whether any representational principle should count as a metarepresentational principle, and whether the contrary is also true. My argument is that it is not the case: only some representational principles are thereby metarepresentational, and not every metarepresentational principle needs to be accepted as a representational principle. In the case of (RE), the substitution of logically equivalent sentences need not ascribe exorbitant logical capacities to the agent, but reflects only the fact the the *description* of her knowledge is likely to be made up to logical equivalence. I think this interpretation is correct, but it faces a number of objections to which I now turn.

3.2 Logical equivalence and distribution over conjunction

One characteristic of **MS** is that the addition of a principle of distribution of knowledge over conjunction results in the closure of knowledge under logical consequence. The addition of :

$$\text{from } \vdash K(\phi \wedge \psi), \text{ infer } \vdash K\phi \wedge K\psi \tag{DIST}$$

yields as a derived rule that

$$\text{if } \vdash \phi \rightarrow \psi, \text{ and } \vdash K\phi, \text{ then } \vdash K\psi \tag{CL}$$

This result is in fact an immediate counterpart to the law: $x \leq y \Leftrightarrow x.y = x$, holding in boolean algebras. It is worth reminding the derivation of (CL) in **MS**:

(1) $\vdash_{MS} \phi \rightarrow \psi$, by assumption.
(2) $\vdash_{MS} \phi \leftrightarrow (\phi \wedge \psi)$, by (1) and CP
(3) $\vdash_{MS} K\phi \leftrightarrow K(\phi \wedge \psi)$, by (2) and (RE)
(4) $\vdash_{MS} K\phi$, by assumption.
(5) $\vdash_{MS} K(\phi \wedge \psi)$, by (3), (4) and MP
(6) $\vdash_{MS} K\phi \wedge K\psi$, by (5) and (DIST)
(7) $\vdash_{MS} K\psi$, by (6) and CP

The fact that this combination of principles results in a form of closure under logical consequence is considered a major shortcoming of Montague semantics. The reason is that the principle of distribution of knowledge over conjunction seems a very innocuous principle as compared to the principle of substitution of logically equivalent sentences. Besides, the principle of closure of knowledge under logical consequence is irrealistic *whether we interpret it as a representational or as a metarepresentational principle.* This principle is certainly not plausible with regard to the way belief contents (however we choose to individuate them) are processed by a human agent; but it is certainly not plausible either with regard to the way we ascribe knowledge. As a consequence, it may seem more reasonable to drop (RE) altogether.[10]

But is the principle of closure of knowledge under conjunction a clear metarepresentational principle? Under a common reading, the principle says that an agent who knows a conjunction should have separate knowledge of both conjuncts. Interpreted in this way, it is an epistemological claim, and the reading is clearly a representational reading. It is precisely in that perspective, for instance, that Dretske ([3]) argues for the fact that epistemic operators are 'semipenetrating', when he writes

> ...it seems to me fairly obvious that if someone knows that P and Q, has reason to believe that P and Q, or can prove that

[10] This conclusion is endorsed by Soames (1987), on account of the above derivation. I note that the principle of distribution is not considered an obvious representational principle by Williamson. See [18], c. 12.

> P and Q, he thereby knows that Q, has a reason to believe that Q, or can prove (in the appropriate epistemic sense of this term) that Q. ... This is not a claim about what it would be appropriate to say, what the person himself thinks he knows or would say he knows. It is a question, simply, of what he knows ([3]: 33).

Dretske is very lucid here in distinguishing between the principle of distribution of knowledge over conjunction as an epistemological claim, and a claim about 'what it would be appropriate to say'. Now, one might argue however that if it is such a clear representational principle, it should thereby be adopted as a metarepresentational principle. This may be a correct metatheoretical principle at first view. Yet it can be challenged in two ways: first I shall argue that the very question whether knowledge distributes over conjunction rests on a semantic-syntactic ambiguity in the first place. Second, I will try to give examples of epistemic statements where the principles (DIST) and (RE) are not both metarepresentationally acceptable, although (RE) is metarepresentationally acceptable, and (DIST) is representationally acceptable.

3.2.1 A syntactic ambiguity

The question of the syntactic ambiguity of knowledge statements is whether an assertion of the form:

(a) X knows that ϕ and ψ [11]
has logical form:
(b) X knows that ϕ and that ψ
and whether this would legitimate in turn:
(c) X knows that ϕ and knows that ψ
so as to yield:
(d) X knows that ϕ and X knows that ψ

I take the step from (c) to (d) here to be entirely non-problematic.

The step from (b) to (c) seems to raise no obvious problem either. Furthermore, this may be what most people have in mind when they argue for the validity of the principle of distribution of knowledge over conjunction.

It is therefore the step from (a) to (b) that seems to be the most problematic in the above schema. There are several ways to contrast (a) and (b). Consider for example Quine's famous example of Ralph seeing Ortcutt under both guises, either as the man seen at the beach, or as the man in the brown hat. Ralph believes of the man seen at the beach that he isn't a spy, whereas he believes of the man in the brown hat that he is a spy. Should we ascribe to Ralph contradictory beliefs? It seems to me that it is plausible to say:

(21) Ralph believes that Ortcutt is a spy and (also) that Ortcutt is not a spy

But it would certainly be odd to say:

[11] I suppose of course that the conjunction is in the scope of 'that', ie that (a) is disambiguated thus : X knows that (ϕ and ψ).

(22) Ralph believes that Ortcutt is a spy and Ortcutt is not a spy

The first statement (21) is clearly of the form (b) above, and the second (22) of the form (a). The reason I think one (at least I) can be reluctant to utter (22) is that it seems we are ascribing an obvious contradictory belief to Ortcutt, whereas in (21), the iteration of 'that' after the conjunction seems to ascribe only nearly-contradictory beliefs to Ortcutt. This analysis is consonant with Quine's remark ([13]: 106), that one may want to say both :

(23) Ralph believes $z(z$ is a spy) of Ortcutt

(24) Ralph believes $z(z$ is not a spy) of Ortcutt

and yet refuse to say:

(25) Ralph believes $z(z$ is a spy and z is not a spy) of Ortcutt

I think Quine's example can legitimate the idea that if we analyze a statement of the form 'X knows that ϕ' as ascribing to X a certain content corresponding to the clause 'that ϕ', then the content corresponding to the clause 'that $\phi \wedge \psi$' need not be the same as the content of 'that ϕ and that ψ'. This is actually the case in Montague-Scott semantics: the proposition expressed by a contradictory sentence, such as 'that $\phi \wedge \neg\phi$', corresponds to the empty set ; whereas the proposition expressed by 'that ϕ and that $\neg\phi$' is the whole set W. We can note further that the distinction between 'that ϕ and ψ' and 'that ϕ and that ψ' is likewise called for in Recanati's theory of belief sentences ([14], c.3). According to him, a belief sentence of type 'X believes that ϕ' is analyzed as: 'X believes that/ϕ', and not as 'X believes/that ϕ' (in compliance with the thesis of semantic innocence, according to which the semantic value of sentences remains the same inside and outside of belief contexts). As a consequence, a sentence like: 'X believes that ϕ and ψ' must have logical form 'X believes that/ ϕ and ψ', which is not the same, without any explicit stipulation to that effect, as: 'X believes that ϕ and that ψ', whose logical form should be: (X believes that/ ϕ) and (X believes that/ ψ).

3.2.2 Knowing and seeing

This remark on the syntax of epistemic sentences does not answer our concern about distribution of knowledge over conjunction. It only enforces the idea that there is indeed a syntactic-semantic ambiguity in the reading of a modal formula of the form $K(\phi \wedge \psi)$. At the level of semantics, it shows at best that one may say of an agent like Ralph, accepting the above reading : *he believes that ϕ and he believes that $\neg\phi$, although he does not believe (that ϕ and $\neg\phi$)*, so that the principle $K\phi \wedge K\psi \rightarrow K(\phi \wedge \psi)$ is not a correct metarepresentational principle. To say that this principle is not a valid principle is generally agreed upon : but what we are looking for is an argument against the converse principle. To see more clearly about distribution over-conjunction, I suggest we draw a parallel between the above derivation of (CL) in **MS**, and a similar derivation one may want to draw using the verb 'see' and logically equivalent predicates. My strategy here is the following: we have seen that, according to Dretske, the principle of distribution of epistemic operators over conjunction is right when taken as a representational principle, and is not thereby a correct metarepresentational principle about 'what it would be appropriate to say'. In [2], Dretske made another distinction, between two forms of seeing (see also [4]): epistemic seeing, and non-epistemic seeing (or simple seeing). According to Dretske, a context such as 'X see...', is referentially transparent, where the complement of the verb is supposed to be an object or event. Yet Drestke acknowledges that in conversational contexts, an ascription like 'Pierre sees an apple' often implicates that Pierre *perceives* the apple as an apple (see e.g. [4]: 99). So here again, we encounter a distinction between a representational and a metarepresentational reading of seeing-sentences. I shall try to show that in the case of 'seeing', the principle of distribution over conjunction is representationally founded, ie when one gives 'see' the purely relational reading. But when it comes to ordinary 'seeing' ascriptions, the principle of distribution breaks down, whereas the correlate of (RE) remains acceptable.

Suppose Pierre is a young kid who sees a square and asserts with pride, pointing to the square: 'this is a square'. Then, it is reasonable to say of Pierre: 'Pierre sees a square'. Now, supposes we formulate for 'see' a principle of distribution (DIST-S) over conjunctive indefinite NPs like 'an AB', where A and B are supposed to be intersective predicates, ie predicates for which it is reasonable to claim that an AB is an A and a B (so this rules out cases like 'a small elephant'). Suppose we apply to 'see' a principle of substitution of logically equivalent predicates (RE-S) analogous to the principle (RE) for knowledge[12]. If we follow exactly the same line of reasoning as above, we then get, informally:

(1) $\vdash$ Every square is a rectangle
(2) $\vdash$ Every square is a rectangle square and conversely
(3) $\vdash$ Pierre sees a square iff Pierre sees a rectangle square, [by (RE-S)]
(4) $\vdash$ Pierre sees a square [assumption]
(5) $\vdash$ Pierre sees a rectangle square
(6) $\vdash$ Pierre sees a rectangle and Pierre sees a square, [by (DIST-S)]
(7) $\vdash$ Pierre sees a rectangle

What shall we say of such an inference? Many people would certainly disagree here as to the validity of this inference, whether 'see' is interpreted as a purely extensional verb or not. I will therefore consider two interpretations of the inference: the first will give 'see' a common metarepresentational interpretation ; the second will give 'see' a purely extensional interpretation supposed to correspond to the genuine representational state of seeing analyzed by Dretske as 'simple seeing'.

Seeing as I think that even in the situation where Pierre, a little kid, wouldn't see the square *as a rectangle*, because he doesn't know what a rec-

[12] In the example to follow, "square" and "rectangle" are logically connected by means of the postulate : "every square is a rectangle".

tangle is, and in the situation in which 'seeing a B' is interpreted as 'seeing a B *as a B*', an ascriber remains perfectly entitled to say: 'Pierre sees a rectangle square', if he is ready to say 'Pierre sees a square'. Again, the former ascription needn't entail that Pierre sees the square *as a rectangle.* It says only that Pierre sees a rectangle-square as a rectangle-square, on the ground that Pierre sees a square as a square. What happens here, I affirm, is only that the content of vision ascribed to Pierre is described in a more complex manner. So I argue that (RE-S) is metarepresentationally acceptable.

Yet, if one keeps the same interpretation, one should infer in the same way, using (DIST-S): 'Pierre sees a square as a square and Pierre sees a rectange as a rectangle', and finally 'Pierre sees a rectangle as a rectangle', which is unacceptable.

The argument boils down to the following: if one interprets 'sees an A' as 'sees an A as an A', then if A is equivalent to AB, it may be the same to say: 'Pierre sees an A as an A' and 'Pierre sees an AB as an AB', altough it shoudn't be legitimate to infer: 'Pierre sees an A as an A and Pierre sees a B as a B'. Here we have a case where the principle (RE-S) is more firmly grounded than the principle (DIST-S).

If this argument is correct, then it should be enough to suggest that in the corresponding derivation of (CL) from (RE) and (DIST) in **MS**, (RE) may be more innocuous than (DIST). My view, once again, is that (RE) and (DIST) are not on a par: one may take (DIST) to be an indispensable representational principle, but it is certainly not a principle governing ordinary ascriptions. In the interpretation given here, we see that (RE) needn't make a distorsion in the putative content of vision of Pierre, whereas (DIST) makes such a distorsion.

Simple seeing There is a second, neutral interpretation of 'see', where the verb is purely relational and extensional, and which corresponds to Dretske's notion of *simple seeing.* In this case, the reasoning can be formalized in first-order predicate logic and shown to be valid. It is worth stressing that no special principle needs to be stated for the conclusion to go through: if 'sees' is an ordinary binary relation, then it trivially satisfies (DIST-S) and (RE-S) in first-order logic. As a consequence, an analysis of epistemic statements that would construe 'X knows that ϕ' as 'X sees a B' in first-order logic would have to buy the conclusion that knowledge is likewise closed under logical consequence, because it satisfies both RE and DIST. This is not the case in Montague-Scott semantics though, where DIST need not be satisfied.

This analogy between the case of seeing and the case of knowing ascriptions should help to understand that there is at least one way to interpret epistemic statements for which the principle of substitution of logically equivalent statements is more innocuous than the principle of distribution of knowledge over conjunction. I think there are good grounds to say that the principle of distribution of knowledge over conjunction is a correct representational principle. But it is not sufficient to count as a principle governing ordinary ascriptions. Likewise, the principle of distribution of seeing over conjunction comes out true in Dretske's analysis of simple seeing, but given that our ascriptions of perception using the verb 'see' tend to give the verb a more conceptual interpretation in most contexts, the principle is not therefore a clear metarepresentational principle.

One important thing may finally be added to thus discussion: the above argument may be simplified if one starts out from the ascription: 'Pierre sees every square' (out of a definite collection). In this case, it is legitimate to infer in first-order logic: 'Pierre sees every rectangle square', but not 'Pierre sees every rectangle and Pierre sees every square' (on the supposition that there are more rectangles than squares to be seen). This argument may be found more convincing in order to see the parallel between Montague-Scott semantics and the case of 'seeing', and it is indeed formally closer: in possible-world semantics, a proposition has the structure of a universal quantifier, and the distinction between 'that $(\phi \wedge \psi)$' and ' that $\phi \wedge$ that ψ' corresponds precisely to the distinction between the quantified expressions 'every AB' and 'every A and every B'. Yet the parallel between the case of epistemic statements and the case of seeing statements would break down in this case: for my purpose was to build a derivation free of logical flaws (such as arise in the case with 'every'), and yet sufficient to elicit a discussion about the acceptability of the conclusion.

3.3 Specific problems in MS

I have argued so far that the principle of substitution of logically equivalent sentences can be entirely ascriber-oriented. In other words, the fact that the ascriber uses equivalent yet syntactically different ways of describing the epistemic content of an agent does not affect what the agent knows or believes *in fact*. I think this view is correct, and I think further that the framework of Montague-Scott semantics is entirely consonant with the discussions of the previous section. Yet some problems remain and I offer to examine into detail how far one may try to defend the principle of substitution of logically equivalent sentences in epistemic contexts as a merely metarepresentational principle.

3.3.1 Necessary equivalence

The main problem faced by intensional logic is that two necessary truthes such as '2+2=4' and '$sh(x) + ch(x) = e^x$' are necessarily equivalent, and true in all the possible worlds. Applying the regularity principle, one should be in a position to ascribe 'John knows that $sh(x) + ch(x) = e^x$' on the basis of 'John knows that 2+2=4'. A symmetric problem arises for the belief of a contradiction. If one believes a contradiction, one should not believe any contradiction, but intensional logic predicts that all logical contradictions express the same proposition.

What about propositional epistemic logic? In propositional epistemic logic, two statements such as '2+2=4' and '$sh(x) + ch(x) = e^x$' should be represented by two distinct atomic symbols p and q. Hence the fact that the two statements are necessarily equivalent cannot result from the mere logical form of their mapping onto these symbols, unless one specifies explicitly that they express the same proposition in our semantics. As a consequence, one should consider the problem of *necessary equivalence* to be distinct from the problem of *logical equivalence.* We will call logically equivalent only sentences which can be computed to be logically equivalent by means of the **MS** rules alone: as a result, two distinct atomic symbols won't be counted logically equivalent.

3.3.2 Logical redundancy

One class of logically equivalent sentences includes the sentences described in (1), where the number of conjunctions or disjunctions may be arbitrary, and the number of negations is an arbitrary even number.

(1) $\phi \leftrightarrow \phi \wedge \ldots \wedge \phi$; $\phi \leftrightarrow (\neg\neg)^{2n}\phi$; $\phi \leftrightarrow \phi \vee \ldots \vee \phi$

By the principle of regularity, one should infer respectively :

(2) $K\phi \leftrightarrow K(\phi \wedge \ldots \wedge \phi)$; $K\phi \leftrightarrow K(\neg\neg\phi)^n$; $K\phi \leftrightarrow K(\phi \vee \ldots \vee \phi)$

Are these inferences problematic? I think a natural answer is to say : 'no'. These inferences can at worse be incongruous. It is incongruous to say 'Pierre knows that it rains and it rains' if one can say: 'Pierre knows that it rains'. One reason to suppose that nobody would make such inferences is not because they would introduce a distorsion in the knowledge content ascribed to the agent, but only because, by the gricean maxim of quantity that recommends not to make one's contribution more informative than required (Grice (1989): 26), it is pragmatically recommended to give an information by the more economic means. However, one shouldn't make any specific proviso concerning the level of description chosen by a specific ascriber in the application of (RE), even though the definition of this level may be given in terms of the syntactic depth of the formula. So doing would indeed turn a pragmatic maxim into a semantic rule. If I say of Pierre : 'Pierre believes that the sky is blue', I can say as well: 'Pierre believes that it is not the case that the sky is not blue'. Making up a special a rule to block the inference would probably be *ad hoc* however. For that reason, I don't think it would be appropriate to try to restrict (RE) by a specification of Carnap's notion of intensional isomorphism (of the sort: 'two sentences are isomorphic iff they have the same syntactic depth and involve the same atomic components').

3.3.3 Complexification

Another problem arises with logically equivalent statements of the form $\phi \leftrightarrow (\phi \wedge \top)$ where $\top$ stands for an arbitrary tautology, as in $\phi \leftrightarrow (\phi \wedge (\psi \vee \neg\psi))$. This may lead to infer from 'Galileo knows that the earth moves' that 'Galileo knows that the earth moves and Bertrand Russell is born in 1872 or Bertrand Russell is not born in 1872'. There seems to be a problem with this kind of inference, namely that it seems to imply that Galileo has some form of acquaintance with the components of the tautology. This however presupposes that a statement of the form: 'X knows that $\phi \wedge \psi$' is in fact read as 'X knows that ϕ and that ψ'. I have argued above that the step from one reading to the other is probably not legitimate however. Hence, this application of the inference rule again should be ruled out merely by an application of the gricean maxim of quantity.

3.3.4 Self-equivalence

One further problem with the regularity principle arises when one starts out from a given logical equivalence between two syntactically distinct formulas ϕ and ψ. The problem is when the equivalence is under the scope of the knowledge operator. If ϕ is logically equivalent to ψ, then $(\phi \leftrightarrow \phi)$ is equivalent to $(\phi \leftrightarrow \psi)$. Now it might be reasonable to say of an agent : 'X knows that $\phi \leftrightarrow \phi$'. But it seems less reasonable to infer: 'X knows that $\phi \leftrightarrow \psi$'. The trouble with this example seems to be that one cannot resort to the gricean maxim of quantity any more to account for the fact that the inference causes trouble in this case. It seems very plausible to say : 'X knows that $\phi \leftrightarrow \phi$, yet X doesn't know that $\phi \leftrightarrow \psi$'.

What then is the difference between this case and the previous ones? The difference here lies in the fact that the form of knowledge here ascribed deals explicitly with an equivalence between sentences. The equivalence is no longer *used* by the ascriber to describe the content of knowledge of the agent, it is *mentioned* as part of this content. This is actually hinted at by the fact that the biconditionnal is iterated in the application of (RE), as can be seen in (2) in the following derivation:

(1) $\vdash \phi \leftrightarrow \psi$, by assumption
(2) $\vdash (\phi \leftrightarrow \phi) \leftrightarrow (\phi \leftrightarrow \psi)$, from (1) and CP
(3) $\vdash K(\phi \leftrightarrow \phi) \leftrightarrow K(\phi \leftrightarrow \psi)$, from (2) and (RE).

Thus, it is very likely that an ascription of the form 'X knows that $\phi \leftrightarrow \phi$' makes some kind of evaluation of X's logical knowledge. I think we tend to say: 'X knows that 'Pierre is nice' is equivalent to 'Pierre is nice' ' more readily than 'Pierre knows that Pierre is nice if and only if Pierre is nice'. For that reason, it is natural to interpret such an ascription as a metalinguistic assertion. Or to use Récanati's terminology, an assertion of the form 'X knows that $\phi \leftrightarrow \phi$' may be systematically read as: 'X so-knows that $\phi \leftrightarrow \phi$'.

3.3.5 Logical truthes and contradictions

$(\phi \wedge \neg\phi)$ is equivalent to $\neg(\gamma \rightarrow (\psi \rightarrow \gamma))$. But it is absolutely unlikely to infer 'X believes that $\phi \wedge \neg\phi$', from 'X believes that $\neg(\gamma \rightarrow (\psi \rightarrow \gamma))$'. The same goes for tautologies. From 'X believes that $\phi \vee \neg\phi$', by (RE) one is in a position to ascribe X a belief expressed by any other tautological expression. This problem seems even more pressing than the previous one, since one may want to make one ascription and deny the other. Note that the case of contradictions is specific of belief contexts, and wouldn't arise symmetrically for knowledge: knowledge being a factive attitude, it doesn't make sense to ascribe an agent a knowledge content expressed by a logical contradiction.

I think there is a natural answer to these problems. It consists first in refusing to ascribe a plain contradiction to an agent. An agent may believe that ϕ, and he may believe that ψ despite the fact that $\psi \leftrightarrow \neg\phi$. In this case, one gets : $B\phi$, $B\psi$, and therefore one can infer $B\neg\phi$. But this doesn't imply that the agent believes $\phi \wedge \neg\phi$. This is exactly consonant with the previous discussion of Quine's example of Ralph. For that reason, I make the supposition that no plain contradiction is ever ascribed to an agent. In Montague-Scott semantics, this proviso will be accomodated by imposing that the empty set is never among the belief set of an agent, even though two complementary sets standing for incompatible propositions may.

One can still object that the agent may give *assent* to some form of complicated antilogy. This objection is not decisive: the failure of an agent to recognize an antilogy is a failure to recognize a *sentence* as expressing a logical truth. It is again a form of metalinguistic belief and should not licence the ascription of any contradiction in the 'that'-clause of the belief sentence.

However, consider an ascription like:'Pierre believes that it is not the case that if Gore is elected, then if Bush is elected, then Gore is elected'. The ascription may be paraphrased: $B\neg(\gamma \rightarrow (\psi \rightarrow \gamma))$ in epistemic logic. Here a classical antilogy is used to describe Pierre's belief. But it seems that the real problem hangs on the interpretation of the conditional, and this problem is clearly more general than what is at stake here. For that reason, I think it is correct to assume that no logical contradiction is ever used to ascribe a belief to an agent, so long as only classical logic is involved in metarepresentational discourse.

The problem remains of what happens when a tautology is under the scope of the belief operator. Here I ask: what does it mean to say of a given agent like Pierre: 'Pierre knows that it will rain or it won't rain'. I think there is some sort of ascription of logical knowledge here. If really I am ready to make such a queer ascription, then I think I should be ready to say as well: 'Pierre knows that 2+2=4 or 2+2≠4', or anything along the same pattern. Certainly, I will refrain from inferring 'Pierre knows that ψ' where ψ is an arbitrary tautology of a greater logical complexity. But the problem we are facing here is clearly of a metalinguistic nature again. The ascription of some form of tautological knowledge to an agent is of a very special form. Here we are dealing with the very logical knowledge of the agent, and no longer with the fact an ordinary piece of knowledge may be *a priori* described up to logical equivalence.

4 Conclusion

The rule of regularity in **MS** is harmless after all, when interpreted as a metarepresentational rule of ascription of belief, and not as a rule supposed to reflect the ascribee's logical acumen. The system **MS** is commonly presented as the weakest sub-normal system of modal logic having a natural possible-world semantics and 'able to avoid all the standard forms of logical omniscience...except one !', namely the rule of substitution of logically equivalent formulae. It is then customary to entertain another solution, which consists in assigning syntactic objects, ie sentences, as the semantic value of the 'that'-clause under the scope of the knowledge verb. Upon what it is customary again to complain that this solution is too drastic after all, for it does not even license the inference from: 'X knows that $\phi \wedge \psi$', to 'X knows that $\psi \wedge \phi$'. Most analyses stop here. Yet why does the rule of commutation of conjunction seem so harmless? A common reply is often that an agent who *assents* to a logical conjunction should *assent* to its commuted counterpart.

I think this quandary rests entirely on a confusion between the level of object-representations and the level of metarepresentations. A natural answer, in my opinion, is to say that we, as ascribers, consider the commutative rule should not make any distorsion with regard to the putative content of the ascribee's belief. We needn't suppose that the agent is capable of assenting to both sentences to adopt this rule. It is us in the first place for whom the equivalence makes sense. Why not keep the syntactic theory though, and simply add the rule of commutation as a sound rule of inference? Well probably because one might have to do the same for disjunction. Then what about double negation? Should we also add the rule: $K(\neg\neg\phi) \therefore K(\phi)$ (supposing a classical construal of negation)?

My view is that a treatment like the foregoing would be consistent, but that it would be hard to say *where* the list of rules should end exactly. For that reason, I think it is methodologically sound to try the opposite: instead of supposing the specification of 'that'-clauses in belief sentences to be very finely individuated, and then adding rules intended to liberalize the range of possible inferences, I favour the move which consists in associating with belief sentences coarser objects, and admitting that the range of possible substitutions is pragmatically filtered out. A rule of inference, to be legitimate, need not be applied systematically. To those who take (RE) in **MS** to ascribe exorbitant logical capacities to the agent, I want to reply the opposite: the range of possible *applications* of a rule such as (RE) in the context of ordinary ascriptions is *de facto* restricted by the set of standard logical equivalences that the ascribers are likely to resort to. Of course this raises the question what that notion of standard logical equivalence really amounts to. This, I think, is clearly a representational problem, and a true problem about logical knowledge.[13]

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