Dummett, Achilles and the tortoise

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DUMMETT, ACHILLES AND THE TORTOISE

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What is the relationship between the meaning of logical words and their use in particular inferences? What is an inference and what does our knowledge of logical deductive rules consist in? How can we be justified in following these rules? Much of Michael Dummett's work has been devoted to an investigation into such questions in the epistemology of logic, and he has illuminatingly shown how they connect with basic issues in the philosophy of language, in the theory of knowledge and in metaphysics. Quite often, his discussions of these issues have taken their start from one or other of the many puzzles and paradoxes which pervade the philosophy of logic more than in any other field of analytic philosophy. I have, however, always been surprised that – as far as I know- he did not discuss at some length, or at least directly, Lewis Carroll's paradox of Achilles and the Tortoise, which is often called the "paradox of inference"[1]. It seems to me to raise many issues which are relevant to his concerns. In what follows, I shall review some of these issues, and I shall try to show that although many of Dummett's views provide an answer to them, it is not so clearly the case for a particular kind of scepticism that one can draw from Caroll's tale, about the normative force of logical reasons.

1. Lewis Carroll's enigmatic piece (Carroll 1895) stages Achilles presenting to the Tortoise three propositions:

(A)Things that are equal to the same are equal to each other
(B) The two sides of this triangle are things that are equal to the same
(Z) The two sides of this triangle are equal to each other

The Tortoise accepts A and B, but does not accept Z, although she[2] accepts the proposition which Achilles presents to her:

(C) If A and B are true then Z must be true

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[1] As it is named, for instance, in a recent collection of paradoxes, Clark 2000. It is not clear, however, that this puzzle is a genuine paradox, in the ordinary sense of a set of acceptable premisses leading to an unacceptable conclusion through apparently acceptable rules of inference, for in the case at point precisely nothing is is inferred from the premisses.
[2] Some writers (Stroud 1969) take the Tortoise's gender to be masculine, others (e.g. Smiley 1995, Simchen 2001) as neutral. My Harrap's Dictionary tells me that it is a feminine noun. Hence I shall adopt the feminine "she".

As Achilles insists, the Tortoise is also prepared to accept that if A and B and C are true, then Z must be true, but she still refuses to accept Z, and the story suggests that the regress never ends.

There are two striking features of this story. The first is that the Tortoise accepts C, and does so because she agrees that C expresses a logical truth.[3] The second is that the Tortoise makes her acceptance conditional on C being entered as a further premiss. The puzzle is: how, given these facts, can she fail to accept Z?

It is not easy to say what the meaning of the tale might be. At least four kinds of morals have been drawn, whether or not they were actually intended by Carroll himself.

1) The most usual moral focuses on the second feature: the fact that the Tortoise needs to add the conditional C as a supplementary premiss shows that she fails to make a distinction between the propositions A and B on the one hand and the logical truth C on the other, and more generally between a premiss and a rule of inference[4]. This is Carroll's own diagnosis when he explains his article to the Editor of *Mind*: "My paradox turns on the fact that in an Hypothetical, the truth of the Protasis, the truth of the Apodosis, and the *validity of the sequence* are three distinct propositions."[5] In other terms, we can neither treat a rule of inference, such as *Modus ponens* (MP), as conditionals, nor as premises. Once we respect this distinction, Carroll's regress cannot start.

2) The second moral has to do with the *epistemology of understanding*. It can be formulated thus : "Understanding and accepting premisses of the form P, and if P then Q, but not accepting the corresponding proposition Q describe nothing at all" (Black 1970: 21) [6]. Either the person in question does not understand the meaning of these sentences or of the words which they contain (in particular the logical word *if*), or her acceptance of the premises is not sincere or faked. The same point can be put in terms of the notion of knowledge: " A man knows that if P, then Q if, when he knows that P, he is able to see that, consequently, Q: If a man knows that P, but cannot see that Q, this is just what shows him not to know that if P, then Q." (Brown 1954: 175). The Tortoise seems to be a person of this kind. Hence she does not know what " if P, then Q" means, or does not understand it.

A more specific way of making a similar diagnosis consists in saying that the Tortoise's predicament illustrates that the kind of knowledge involved in this case cannot be expressed by a *proposition* like C. Our knowledge of the rule is not a form of knowledge *that* or propositional

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[3] See Simchen 2001, The Tortoise agrees that (Z) "follows logically " from (A) and (B), and that "the sequence [expressed by (C)] is a valid one" (Carroll 1995: 691)

[4] Among the reactions to Carroll's paradox along these lines, see Peirce 1902 (*Collected Papers* 2.27), Russell 1903, p. 35, note, Ryle 19 45-46, Brown 1954,Geach 1965, Thomson 1960, Smiley 1995

[5] Dogson 1977, p.472 Quoted by Smiley 1995

[6] Quoted by Stroud 1969: this is also, basically, the moral that Stroud derives from the paradox.

knowledge but a form of knowledge *how*, or of practical knowledge. This was indeed Ryle's reaction to the paradox, closely related to his 1954 article mentioned above[7].

One could also read the tale as a version of Wittgenstein's "scepticism" about rules, as it is interpreted by Kripke (1981). What the Tortoise's refusal to accept the conclusion suggests, on this view, is that there is no principled reason to interpret the sequence A-B-Z as an ordinary instance of *modus ponens* rather than as an instance of a deviant rule of *schmodus ponens* (e.g "from P and if P and Q infer Q, unless P and Q are propositions about geometry, in which case do not accept Q"). Of course given that neither Carroll nor the Tortoise had heard about Kripke, this interpretation is far fetched, but there are clear similarities with Wittgenstein's problem. Wittgenstein points out that following a rule is not an interpretation. Adding successive, possibly indefinite, interpretations of the rule as conditionals cannot fix its meaning.

3) The third kind of interpretation concerns the *justification of logical laws*. Although it is clear that this is an extrapolation from Carroll's tale, we could read it as making a point about the justification of logical rules: suppose that logical rules could be justified by the fact that we have accepted general conventions. Suppose, in particular that such conventions took the form: for any x, y , and z, if x and y stand in the MP relation to z, then x and y imply z. Now suppose that the particular sentences 'P' and 'If P then Q' stand in the MP relation to 'Q'. Do 'P' and ' if P then Q' imply 'Q'? If we reason thus :

(a)'P' and ' if P then Q' stand in the MP relation to 'Q'
(b) if ' P' and ' If P then Q' stand in the MP relation to 'Q', then 'P' and ' If P then Q' imply 'Q'
(c) hence 'P' and 'if P then Q' imply 'Q'

But the question whether (c) can be inferred from (a) and (b) turns out just to be the question whether it is true that for any x, y , and z, if x and y stand in the MP relation to z, then x and y imply z. In other words, if we want to explain the nature of logical truths or rules from the existence of conventions, we must presuppose these logical truths and rules to derive them from conventions, and our derivation is thus circular. This is indeed Quine's famous argument against Carnap's conventionalism about logic in "Truth by convention". Quine explicitly refers to Lewis Carroll's regress when he mounts his argument. [8].

A stronger reading along these lines would be that the Tortoise is not so much rejecting a particular account of the justification of logical rules than expressing scepticism about *any* sort of

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[7] Ryle 1945-46, Ryle 1949
[8] Quine 1936, in (1976), p.104, note 21. The presentation above is from Harman 1996 p.395

justification of them . As we have already noted, she neither disputes the validity of the "sequence" C nor the validity of the inference from A and B to Z. But her refusal to accept the conclusion C might be the expression of her scepticism about logical truths or logical rules in general. In other words the Tortoise would be a radical sceptic about deduction, just as Hume is sceptic with respect to induction.

4) The fourth interpretation, unlike the others, does not focus on the second feature of the story – the insertion of an additional premiss -   but on the first: the Tortoise *accepts* C. Given this why does not she accept Z? Again she perfectly agrees that C is a logical truth. So the problem is : how can someone who accepts a logical truth or a logical rule fail to infer accordingly? One answer could be that she actually does not really accept the logical truth C, either because she does not really understand the meaning of C. This would bring us back to interpretation 2). Another answer would be that the Tortoise accepts C, fully understands what it means, but still *cannot*, or possibly *refuses*, to draw the appropriate conclusion. She would thus intend to convey another form of scepticism, which would bear upon about the *normative force* of the logical laws or rules. On this reading the Tortoise is fully aware that C is a logical truth or a valid rule of inference. But she fails to draw the conclusion because she does not take the rule or law as binding and as being able to *move* her to the appropriate conclusion. Either she is a sort of akratic in the domain of logical inference, seeing what she ought to infer, but failing to comply, or she explicitly questions the normative power of the *logical must*. The Tortoise recognizes that Z follows from A and B by "logical necessity". She recognizes that "any one who accepts A and B as true, *must* accept Z as true", and that "The youngest child in High School…will grant that." For all that, she still refuses to accept Z, and challenges Achilles "to force [her], logically, to accept Z as true." This is why her acknowledgement of the authority of logic is somewhat ironic: "Whatever *Logic* is good enough to tell me is worth *writing down*." She writes it down, but does not act accordingly. Hume is often presented as a sceptic about practical reason: reason, or belief, as such, cannot move us to act, only passions or desires can do so.[9] In parallel fashion, the Tortoise would be a sceptic about the power of logical reasons to force us to believe any sort of conclusion. She would be a sceptic about *the force of logical reasons*. The question is: "How can logic move the mind?"[10]

2.   Although there are very few allusions to Carroll's paradox in Dummett's writings, it is obvious that they are relevant to a number of the issues just mentioned. Dummett's diagnosis is very much along the lines of 1) above:

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[9] See Kosgaard 19XX
[10] Blackburn 1995, Engel 1998.

"In *Principles of Mathematics*, Russell falls into confusion through a desire to say both that e.g, 'Peter is a Jew' is the same proposition when it occurs alone and when it occurs in 'If Peter is a Jew, then Andrew is a Jew', and that it is not. It must be the same, because otherwise modus ponens would not be valid; it cannot be the same, because then 'Peter is a Jew; if Peter is a Jew, Andrew is a Jew; therefore Andrew is a Jew'' would be the same as 'If both Peter is a Jew and if Peter is a Jew, then Andrew is a Jew, then Andrew is a Jew' and it was precisely Lewis Carroll's discovery (in 'What the Tortoise said to Achilles') that it was not." Frege provides a solution by saying that the sense of the two occurences of 'Peter is a Jew' (the thought expressed by them) is the same, but that the assertoric force is present in one and lacking in the other.(Dummett 1973: 303)

 There are actually two points here, although intimately related. The first is what Geach (1965) calls "the Frege point" : that one should distinguish two occurrences of the same sentence when the one as asserted and the other not (hence that the assertion sign, and the assertoric force that it conveys, should be applied to a sentence as a whole, and not to its constituents[11]. The second is that one should distinguish a *logically true proposition*, such as "If A, and if A then B, then Z" not only from a premiss in an argument, but also from a *rule of inference*. The regress cannot start if one recognises that A and B do not have the same status as C.[12] These distinctions are undoubtedly of first importance, and Dummett emphasises them in many places, for instance when he contrasts Frege's conception of logic as a science of *truth* with Gentzen's conception of logic as a science of *inference*.[13]

They are also quite relevant to Dummett's (1973a, 1991) treatment the problem of the justification of deduction, which can be considered as a direct answer to the sceptical Tortoise 3). Dummett characterises the problem of deductive inference through

"a tension between what seems necessary to account for its legitimacy and what seems necessary to account for its usefulness. For it to be legitimate, the process of recognizing the premises as true must already have been accomplished whatever is needed for the recognition of the truth of the conclusion; for it to be useful, a recognition of its truth need not actually have been accorded to the conclusion when it was accorded to the premises. (1973a: 297)

 As Dummett notes further, "on the ordinary notion of proof, it is *compelling* just because, presented with it, *we cannot resist* the passage from premises to conclusion without being unfaithful to the meanings we have already given to the expressions employed in it."(*ibid* 301, [my italics]). Carroll was certainly familiar with Mill's view, discussed by Dummett, that deductive inference is a *petitio*

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[11] This is also Russell diagnosis: according to him the solution to Carroll's puzzle resides in the distinction between the relation of (implies) which holds between two propositions, and which can be considered without being asserted, and the relation of consequence (*therefore*) which holds between two *asserted* propositions. Russell explicitly mentions Frege in this respect.

[12] Ryle 1954, although he notoriously takes the *modus ponens* rule to be an "inference ticket" to the conclusion, to which Geach (1965) objects that if the inference from the premisses to the conclusion is valid, there is neither need of a supplementary premiss nor of a licence for going from the premises to the conclusion.

[13] Dummett, 1973: **add ref**, 1991 **add ref**

principii and hence useless. Thus the Tortoise's refusal to draw the conclusion might be interpreted as an acknowledgement of the tension between the usefulness and the legitimacy of deductions. Dummett describes two views which attempt to solve this problem. The first is Wittgenstein's, who, in claiming that when we move from premises to conclusion we are giving a new meaning to the expressions we employ in it, denies that the connection between premises and conclusion is necessary and therefore that the proof is compelling. Again Carroll's Tortoise had not read Wittgenstein, but her reluctance to grant the "hardness of the logical must" is strikingly similar: she denies that "logic is going to take you by the throat and *force* you to do it". The second view is the holistic conception, according to which deduction is justified, but only against the global background of our inferential practices. Justification, on this view, can only be circular[14]. This is quite in line not only with Quine's argument in "Truth by convention", but also with his holistic conception of meaning. Dummett explicity rejects both. Quine's refutation of Carnap's conventionalism does not involve any distinction between logical laws and rules[15], and his own holistic conception of the justification of logical laws prevents him from making it:

> "Quine's thesis [that nothing is immune to revision] involves that the principles governing deductive connections themselves form part of the total theory which, as a whole, confronts experience. Presumably, in order to avoid Achilles and the Tortoise troubles of the Lewis Carroll type, we must recognise the total theory as comprising rules of inference as well as logical laws in the form of valid schemata and their instances: but there is no reason to suppose that Quine draws a distinction between the status of such rules as against laws like Excluded Middle; they too must be equally liable to rejection under a heavy enough impact from without." (1973a: 596)

Justification can be justified only on the basis of a conception of logic in which the primary concept is that of inference and not that of logical truth, and on the basis of a molecularist theory of meaning according to which each sentence possesses a determinate content which can be grasped independently of our knowledge of the whole language. For logical constants, this implies that their meaning be determined directly by the deductive rules associated with them, and that they satisfy the requirement of harmony and conservative extension. When we derive a given logical law from others, we have what Dummett calls a "proof-theoretic justification of the first grade". But such a justification is only "relative", for the derivation already assumes the validity of certain other laws (or, in a justification of the form (a)-(c) above, of the very same law that is meant to be justified). In order to have a non relative justification, certain laws must be "self-justifiying", and they are such

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[14] Goodman and Rawls' notion of "reflective equibrium" would be here the appropriate kind of justification, as Dag Prawiz reminded me. See his 1978 on this point.
[15] See Tennant 1986: **add ref** .Quine presentation of the regress of justification to which the derivation of logical truths from conventions leads explicitly employs axioms and their instances. Tennant argues that once *modus ponens* is conceived as a natural deduction rule (conditional elimination); the regress cannot start..

when they satisfying the requirement of harmony. [16] Dummett's views have attracted a lot of discussion[17], and it is clear that if the main issue raised by Carroll's paradox is that of scepticism about the justification of deduction, they are among the most forceful responses to this challenge.

I do not, however, intend to discuss here Dummett's conception of the justification of deduction. For I do not believe that an answer to a sceptic's challenge about justification can provide an answer to the other form of scepticism raised by Carroll's paradox, i.e 4), about the force of logical reasons. For this scepticism can arise even when we have a philosophically satisfactory account of the justification of logical rules. There are three things to distinguish in the story[18]:

(1) Do the two premises A and B imply the conclusion Z?

(2) Given that one has good reasons to believe that A and B are true, is it reasonable to believe that Z is true?

(3) Supposing it is reasonable for me to believe Z on the basis of A and B, what is supposed to move our mind to believe that Z ?

(1), as we saw, is not in question. Neither is (2), if we are equipped with an appropriate – let us suppose - molecularist justification of our rules of deduction. The point of scepticism about the force of logical reasons is that neither a positive answer to (1) nor a positive answer to (2) can yield a positive answer to (3). However willing to recognize the validity of the inference, and however well equipped with a full justification of deduction in general, this kind of sceptic will not be moved.

As I have already suggested above, in order to understand the challenge, it is interesting to compare it to a parallel kind of challenge for practical reasoning. Simon Blackburn (1995) in particular, in his essay for the centenary of Carroll's piece in *Mind*, shows that a similar problem to Carroll's arises when we consider an instance of practical reasoning:

(P) I prefer lettuce to souvlaki

(B') The moment of decision is at hand

(Z') Let me choose to eat lettuce rather than souvlaki

How many premises of the form

(P*) It is right (good, desirable, rational, etc) to prefer lettuce to souvlaki

are added, the Tortoise still does not act. Blackburn argues for a "Humean conclusion" to the problem of scepticism about practical reasons: "There is always something else, something that is not under the control of fact and reason, which has to be given as a brute extra, if deliberation is ever to end by determining the will." (1995: 695).

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[16] Dummett 1973 a, 1991, ch. 10, 11
[17] e.g Haack 1982 , Tennant 1986, Engel 1991
[18] see Schueler 1995

There are, of course, strong differences between practical reasoning on the one hand, and logical reasoning and theoretical reasoning on the other hand: on the standard account, the former goes from beliefs and desires to intentions or actions, whereas the latter goes from beliefs to beliefs; desires do not have the same "direction of fit" as beliefs and these are not, in general, under the control of the will, unlike actions. But the form of the sceptic's challenge about the force of logical reason is very close to the one which affects the practical case. For suppose that the Tortoise had accepted not only C but the apparently stronger:

(C*) It is a norm of correct deductive reasoning that if one accepts that MP kind of inferences
       are valid, one should in the presence of a particular instance of MP, accept the conclusion

Suppose also that the Tortoise had recognised that

(I) The propositions A, B and Z stand in the MP relation

Wouldn't she be bound to accept Z ? No. (I) here plays the same role as the minor premiss of the practical syllogism (P)-(Z') above. But even though the Tortoise sees its truth, and accepts (C*), she is not be moved. Neither would she if we reinforced again (C*) into :

(C**) There are absolute rational justications for the norm of correct deductive reasoning
        that if one accepts that MP kinds inferences are valid, one should in the presence of an
        instance of MP, accept the conclusion

and so on. Whether we say that our norms of reasoning are justified because they are metaphysically necessary, or because they correspond to what ideal agents would believe in ideal circumstances, they would not be able to *force* us to reason according to them.

It is important to distinguish this (type (4)) scepticism raised by the Tortoise's resistance to accept the obvious conclusion from the type-(3) scepticism about justification. The sceptic about the force of logical reasons grants that we do have good reasons and justifications to infer according to a given rule of inference. Following the terminology which is common in the literature on practical reasoning, let us call these reasons *normative reasons*.[19] Now to say that they are normative is to say that, at least *prima facie*, they are such that a subject who recognises them as reasons would be ready to act, or in the case at hand, to infer, according to them. A subject can, however, have good reasons to do an action, or to have a certain belief, but still not act upon this reason, and act upon other reasons, which might not be good ones. Such reasons will explain our action, and motivate it, but they will not necessarily coincide with the normative reasons that we have. They can be called *motivating reasons*. Of course the distinction between normative and motivating reasons does not imply that they cannot be identical. In the normal case, when, so to say, everything goes right, the

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[19] See for instance Dancy 2000. For a more elaborate parallel between the practical case and the theoretical case, see Engel to appear.

reasons *for* acting or believing in a certain way just are the reason *why* the agent acted or believed in that way. But the two can diverge. Cases of *akrasia* are a well known example where normative reasons fail to be motivating, but there are simpler examples of the distinction. To use a well worn example, I have reasons not to drink the glass of transparent liquid that is before me, because, unbeknownst to me, it is a glass of petrol, although I have a reason to drink it if I believe that it is a glass of water. In the case of having reasons for inferring a certain conclusion from certain premises which concerns us here, the situation is similar. Someone who believes that P and believe that if P then Q, has a normative reason to believe that Q – since the latter belief follows logically from the two others - but he still might not believe that Q. For he might, for other reasons, happen to believe that not Q (for instance he has sworn to his mother that he would never believe Q). He might also believe that Q, as expected, but for other reasons, completely unconnected with P (Q is his favorite proposition: every time he encounters it, he asserts it). The *modus ponens* rule is a proposition of logic. As such it is worth being written down. But he does not tell you in itself anything about your beliefs. It tells you that if P, and if P then Q imply Q, but it does not tell you that if you *believe* that P, and if you believe that if P then Q, then you *believe* that Q.[20] Similarly when we open a logic book, we just see a number of propositions written. The fact that they are in the book does not in in itself move us to reason logically.We might nevertheless want to formulate the MP rule as a normative requirement of reasoning and belief. Then we would formulate it as a normative reason for belief, using the deontic term "ought":

     (O1) You ought (to believe Q, if you believe that P, and that if P then Q)

This is just a reformulation of (C*) above. But the "ought" which figures in this proposition has a wide scope: it governs a conditional. (O1) does not allow you to infer or detach that

     (O2) You ought to believe that Q [21]

for the reason that we have just noted . Your normative belief O1 does not tell you what particular belief you have to adopt relatively to Q, it just tells you that your recognise O1 as a normative reason.On the present interpretation of the paradox, the Tortoise intends to remind us of this simple, but very important, point.

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[20] Harman 1986 makes this point.
[21] Broome 2000, 2002 emphasises this point.

If we take this lesson of Carroll's tale seriously, the important question becomes: what is the relationship between the normative and the motivating reasons when it comes to having particular beliefs? Normally we should expect the normative reasons to be in some sense able to move us to having certain beliefs and to make certain inferences, that is to enter into certain kinds of psychological states. It is natural, therefore, to think of this relationship as a causal one. But how can normative reasons, which are supposed to be objective and independent of the agent's specific mental states, be able to move our minds in a particular way? [22]

3. If we interpret Carroll's story in this way, what kind of answer shall we adopt? The answers lie between two extremes.

One extreme view consists in adopting a form of Platonism about logical reasons. It was Frege's view: once we grasp that a certain proposition is a truth of logic, a "law of Being-True", we cannot but infer accordingly. Logical laws *as such* are normative, both in the sense that they are absolutely correct, but also in the sense that they motivate us to infer in conformity with them. Being eternal laws of truth, the logical laws do not tell us anything about our psychology or about mental representations. But we have nevertheless a certain kind of relationship with them, which Frege, as most Platonists, conceives as a kind of intuitive grasp (*Fassung*). [23] So it is the direct, intuitive apprehension of the validity of *modus ponens* that is sufficient to move our mind to infer accordingly. In the field of practical rationality, this kind of view has been revived by writers who construe normative reasons as certain objective *facts*, the intuitive recognition of which is supposed to move directly the agent into doing the appropriate action. The view can be transposed to logical reasons. [24] The obvious problem with this view is precisely the one that the Tortoise, on our present interpretation of the story, is meant to remind to Achilles (who, for that matter, seems to be a straight Platonist): a Fregean Thought or a Platonic fact does not have any causal impact upon our minds, and intuition of Platonic fact seems even more mysterious than the idea of a causal contact with Laws of being true and our minds.

The other extreme view lies at the opposite end. It says that only a *psychological* state can play the *causal* role of moving the mind. This is an equivalent of the Humean solution that Blackburn

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[22] Simchen 2001 formulates well, this form of scepticism, but his analysis of the paradox focuses on the question whether a rule can be action guiding, especially when it is quoted. I cannot deal here with this specific interpretation of the paradox.

[23] In the Prefqce to the *Grundgetzetze der Arithmetik*, Frege tells us that we should distinguish two senses of "law" when we talk about the laws of thought: the normative and the causal. Psychologism makes a confusion between these two senses. For Frege, the laws of thought are normative in so far as they are descriptive of a Third Realm. See Dummett 2Frege's Muyth of the Third Realm" in Dummett 1991a .

[24] See Engel *to appear.* The Platonist view in the practical case is defended by Dancy 2000.

suggests for the practical case. On such a view the existence of objective reasons for inference will never "move the mind". The mind, when it comes to ending a deliberation by an action, or a reasoning by the assertion of a conclusion, is always moved by some "brute extra". In the case of action, it must be, for a Humean, a desire or a passion, which is the only psychological state which can motivate one to act. In the case of reasoning from beliefs to beliefs, it has to be also some psychological state. But it cannot be a belief, for beliefs, at least on the official Humean doctrine, do not in themselves cause us to act. They cannot be motivating reasons. Hence the "brute extra" must consists in something else. The most plausible view would be that it is a *habit*. We could understand this in the way Hume gives a "sceptical solution" to his sceptical doubts about induction, by pointing out the force of habit and custom, or in a Wittgensteinian fashion, by saying that at some point, reasons must come to an end, and that drawing a conclusion from premises according to a rule of inference is "simply what we do". It is a brute fact of our practices that we reason *that* way.[25]

The problem with this is that it seems to give us a plausible answer to the causal question about the normative force of reasons at the expense of the justificatory nature of reasons. We lose sight, on this view, of why logical reasons are good reasons, and of how they can be objective. As Dummett says, to be told that we have the practice or habit of inferring in a certain way, and that *modus ponens* kinds of inferences belong to these practices, does not tell us whether we should maintain this practice if we believed the theory according to which the force of reasons consists in nothing but the existence of this practice.[26] Wittgenstein's kind of answer consists simply in questionning the normativity of logical reasons. He says, famously:

« F.P. Ramsey once emphasised in conversation with me that logic was a 'normative science'. I did not know exactly what he had in mind, but it was doubtless closely related to what only dawn on me later: namely that in philosophy we often compare the use of words with games and calculi which have fixed rules, but cannot say that someone who is using language *must* be playing such a game."(*Philosophical Investigations,* 38)

But this leaves us with a mystery: why is it that we find the reasons for our beliefs, and especially our inferential reasons, satisfactory and even compelling? I agree here with Dummett when he says, about Wittgenstein, that "we ought not to rest content with saying, of any feature of our linguistic

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[25] This would in some sense harmonise with what Kripke (1981) calls Wittgenstein 's sceptical solution" to his "sceptical paradox" about rules. Dummett hints at this "brute fact" when he describe Wittgenstein's position: "We do, by and large, agree on what the consequences are, what follows from what, what is a valid proof and what is not…On Wittgenstein's view, it is a *brute* fact: nothing explains it." ("Wittgenstein on Necessity: some reflections", 1993:449). On Wittgenstein's views on logic, see also Lear 1982 : 387 (who defends an interpretation quite distinct from Dummett's), and Railton 1999
[26] Dummett "Wittgenstein on necessity: some questions", in Dummett 1993 : 446

practice, "That is simply what we do".[27] Not only does this view deprive logical reasons of any

normativity, but also it does not tell us what they *do*.

But there are intermediate answers between these two extremes. The most natural one, if we

agree that there must be some psychological state to cause the mind to infer a conclusion, would be

that the state in question is a *belief* about the validity of the rule. But it is precisely the view which is

the target of Carroll's paradox. Let us call this *reflective internalism*.[28] According to it, the subject

would be reasoning by entertaining beliefs to the effect that a given rule is valid, and that he is

presented with a particular instance of this rule, by reasoning in the following way:

  (i) Any inference of the form MP is valid
  (ii) This particular inference (from A and B to Z) is of MP form
  (iii) Hence this particular inference (from A and B to Z) is valid

Not only such beliefs would be reflective, in the sense that they would be beliefs about beliefs, but

they would also be inferential, since (i)-(iii) is an inference. But the point of Carroll's story is that

this kind of reflective belief would lead us to the familiar regress. This does not mean that we can't

have, when we reason from P to Q, such beliefs as *that if P were true , Q would be true*, or *that P

implies Q*, but only

that even if the person is said to believe or accept some statement R linking things he already believes
with his conclusion, we still must attribute to him something else in addition if we are to represent his belief
in that conclusion as based on those other belief. The additional factor cannot be identified as simply some
further proposition he accepts or acknowledges. There must always exist some "non propositional" factor if
any of his beliefs are based on others." (Stroud 1979: 189)

Here Stroud's diagnosis is quite close to Blackburn's Humean view. He does not dispute that some

beliefs are based on others, nor that there can be a "linking" belief of the reflective kind[29], but denies

that the belief in question is part of the content of the reason. Reflective internalism is the

psychological counterpart of the Carroll story. Reflective beliefs about the validity of our inference

rules can no more move our minds than the fact that further premisses are added to the initial ones in

the tale. As Paul Boghossian says, discussing this point:

"At some point it must be possible to use a rule in reasoning in order to arrive at a *justified* conclusion,
without this use needing to be supported by some knowledge about the rule one is relying on. It must be

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[27] Dumett 1973: 309.
[28] The view describe is quite close to what Boghossian (2003) describes as "simple inferential internalism" (2003, see
also his 2002 article, where adopts slightly different formulations). I use here only *partly* Boghossian's formulation for
he is concerned, in his paper, with the question of the *justification* of the rules of reasoning, and not with the question of
their normative force.
[29] The notion of a "linking" belief is also the one used by Broome *op.cit.*

possible simply to move between the thoughts in a way that generates justified belief, without this movement being grounded in the thinker's justified belief about the rule of the reasoning (Boghossian 2002:37)

Reflective internalism says that our belief in the validity of the rule is the product of an inference, (i)-(iii). But we might still accept the idea that we must have in some sense an access to the thought that the inference is valid without this access being *inferential* or even reflective. We could say that this access is a direct access, which allows us to go from premises to conclusion through some insight or intuition, rather than through an act of reflection and an inference. For instance, Laurence Bonjour describes it in the following way (about disjunctive syllogism, but the latter is of course equivalent to MP in the form *not P or Q, not not P therefore Q*) :

"I am invited to assess the cogency of inferring the conclusion that David ate the last piece of cake from the premises, first, that either David ate the last piece of cake or else Jennifer ate it and, second, that Jennifer did not eat it…The obvious construal of this case from an intuitive standpoint is that if I understand the three propositions involved, I will be able to see or grasp or apprehend directly and immediately that the indicated conclusion follows from the indicated premises, that is that there is no way for the premises to be true without the conclusion being true as well. It is obvious, of course, that I might appeal in this case to a formal rule of inference, namely the rule of disjunctive syllogism. But there is no good reason to think that any such appeal is required in order for my acceptance of the inference as valide to be epistemically justified. No…is there any reason to think that such a rule would not itself have to be justified either by appeal to the same sort of apprent *a priori* insight at a more abstract level or else to other rules or propositions for which an analogous sort of justification would be require." (Bonjour 1998: 105-106)

Here Bonjour insists on the fact that rational insight does not depend "on any further ratiocinative process", nor on any reflective apprehension of the rule. Let us call this *non reflective internalism.* Of course, in our post-Wittgensteinian times, a likely reaction to such a proposal may be to reinstate Wittgenstein's qualms about intuition: if it can lead me in one way, it can lead me in another, and it does not give me the possibility to distinguish between what seems right and what is right. But the proponent of the idea of rational insight has a point, if he means that the kind of mental state involved in the recognition of a certain pattern as instanciating a valid is neither reflective nor inferential: if it is not, then no regress like the Carrollian one does not seem to arise. [30] We may

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[30] Crispin Wright suggests something of that sort in his commentary of Boghossian 2002 (Wright 2002: 83). He calls it « simple internalism ». Wright also suggests (p.78-79) that the regress as presented by Boghossian might not be vicious but harmless. I cannot deal with this point here, which has more to do with the problem of justification , which is not my direct concern.Something like this view is suggested by Bill Brewer. " There is more to grasping the laws of logic or mathematical argument than simply being disposed to have one's beliefs mirror the moves they prescribe. Epistemologically productive reasoning is not a merely mechanical manipulation of belief, but a compulsion in thought *by reason*, and as such involves conscious understanding of why one is right in one's conclusion. (Brewer 1995: 242) Brewer talks of « causation in virtue of rationalisation ». But this seems to me just to give a label to the problem, not to give a solution ot it.

grant this point without finding this kind of description of what happens when one makes simple inferences less problematic than the Platonist's appeal to a faculty of intuition.[31]

None of these solutions – Platonism, Humeanism, reflective internalism, non reflective internalism– is satisfactory. But the following points emerge. First, reasoning, in the deductive manner, from premisses to conclusion does not involve any sort of belief about the normative rules that one's follows. When we reason, we do not move from beliefs to beliefs through some linking proposition *about* the link between our beliefs, but we move directly from the content of certain beliefs to other contents. In this sense, our reasoning (at least for simple kinds of inference) is non reflective and "blind"[32]. But second, for all that, our reasoning is not blind, in the sense in which it would be simply a causal process leading us from certain beliefs to other ones. It has to be, in some sense, the exercise of a rational capacity, not a brute causal fact.

4. Dummett's approach to the issues raised by Carroll's paradox as I described them in the previous section has been only indirect. It is not difficult to see why. In the first place, the chief problem which interests Dummett is that of the justification of deduction. It is then normal that he has no interest for the problem about the *force* of reasons as I have formulated it. In the second place, Dummett would probably remind me here about Wittgenstein's distinction between reasons and causes. The problem of finding an appropriate causal factor which would play the appropriate explanatory role in an account of how minds can be moved to infer would probably seem to him to imply some form of category mistake or a reversion to psychologism. Actually I do not think that psychologism is *always* irrelevant in the philosophy of thought, but this is not here the place to argue for this. [33] But I am sure that Dummett would agree that we must at least give some plausible philosophical account of the kind of knowledge which is involved, which would allow us to understand not only why the Tortoise is justified to infer her conclusion, but also compelled to do so. And indeed Dummett's work gives us a number of clues to answer these questions.

It seems quite clear that the natural place to look for an answer to our problem is in an account of the way in which our *understanding* of the premisses relate to our inferring the conclusion, as the second moral that we drew from the paradox suggests. In the first place, as Stroud (1979) has shown convincingly, Carroll's puzzle reminds us that to actually believe a certain proposition, especially on the basis of others, is not to have more propositions "written", either on

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[31] Boghossian 2003: 235-236.
[32] Boghossian 2003, an elaboration of Wittgenstein's famous remark: "I obey the rule blindly"
[33] See Engel 1996, and Engel 1998a

one's notebook on in some place in our minds.[34] It requires understanding it, where understanding is a complex kind of state. Our paradox obviously turns upon this, as Black's quotation above reminds us. If the Tortoise really understands the meaning of *if*, it seems absurd to say that she fails to perform a simple MP kind of inference. But precisely, in what consists the meaning of a logical constant such as *if*? But, for the very same reason that believing that P is not simply to possess some sort of representation of P stored somewhere in a place where we could retrieve it, but actually understanding it, to have a certain account of the meanings of logical constants will not be sufficient to provide a speaker with an adequate understanding of their meanings, at least if this account is formulated in a certain way. Dummett has forcefully argued, in particular against Davidson's truth-theoretic version of what a theory of meaning could be, that such a theory, conceived as an abstract representation of the truth conditions of a language, would not suffice to give us genuine, "full-blooded" understanding of this language. A distinction which the Tortoise needs is that between knowing that a sentence is true and knowing what proposition the sentence expresses. In this sense, she might be said not to understand the sentence C that she adds to her premisses. Similarly knowledge of what a sentence *s* means cannot simply consist in knowing a set of T-sentences of the form "*s* is true iff P", for one must also understand the biconditionals themselves. A regress here looms too. [35] But what would the Tortoise understand if she knew what the words presented to her mean? Does it consist in the canonical rules of inferences associated with the words? Dummett's molecularism rests upon a view of this kind, when he associates the assertion and justification conditions of logical constants to their meaning. Other writers, such as Peacocke and Boghossian have proposed versions of the view according to which the meaning of logical constants is constituted by their canonical inferential rules.[36] But, as I said above about the problem of the justification of deduction, I do not think that a theory about the meaning of logical constants and a theory of the transfer of warrant from premisses to conclusion can *by itself* solve the problem of the force of logical reasons. For the latter problem, to repeat, is not the question whether we are justified in our inferences, but the question of *how*, given that we are justified, we can account for our *actually* performing the inferences in question. Certainly the *shape* of the account of the meaning of logical words that we give affects the kind of account that we can give of how logic can move our

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[34] Dennett 1978, Stroud 1979: 194. A certain verison of the thesis that there is language of thought is open to the Carrollian objection.
[35] Dummett , "What is a Theory of Meaning" (I), 1975 and II (1976)
[36] Peacocke 1992, Boghossian 1999, 2002, 2003. For an opposite view, see Williamson 2003

mind, but it is certainly not sufficient.[37] So let us suppose that an account of logical constants is correct. It still would not tell us how one infers.

The obvious suggestion to answer this question is to try to characterise the kind of knowledge that one can have of a word like *if*. And the natural answer would seem to be Ryle's, *i.e* that it is a piece of practical knowledge, of knowledge how, and not of propositional knowledge, knowledge that. This quite of knowledge would, moreover, be implicit or tacit, and not explicit. But, as Dummett has pointed out quite often in his discussion about knowledge of language, although the kind of knowledge that we have of the rules which govern the words of our language (and therefore of logical words as well) is implicit in the sense that it manifests itself partly through a form of practical ability and partly by readiness to acknowledge its content when presented, neither it is correct to characterise it as a form of practical knowledge nor is it sufficient to characterise it as a form implicit knowledge. [38] Knowing a language, Dummett insists, is neither like knowing how to swim nor like knowing explicitly a set of propositions. To say that it is implicit knowledge in the sense of it being susceptible to conscious access in certain circumstances does not tell us much, unless we can state how this access can occur and how the knowledge in question is applied. For the same reason, Dummett considers that characterisation of our knowledge of meaning in terms of tacit knowledge in the psychological style is just idle, and does not nothing to explain how it is delivered – how possession of the unconscious knowledge operates to guide, prompt, or control the speakers utterances.." (1993: xi, 101; 1991:97).

Dummett tells us that it is not enough to say that knowledge of meaning is implicit. It has to be manifested in the *use* that speakers make of the language. But this "manifestation requirement" is again in itself not sufficient. For the use in question must be such that the speaker is also *aware* of what this use amounts to. Dummett tells us that being aware of the meaning of one's word does not necessarily involve "having it in the forefront of one's consciousness", but this kind of knowledge must involve some sort of recognitional activity. What kind of recognition? The obvious answer is: recognition of fulfilment of the condition which establishes that a given sentence is true.(1991: 317 1993: 45). In other words, recognition of the assertability conditions of the sentence. On Dummett's

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[37] For instance Boghossian (2003) gives the following account: " A deductive pattern of inference P may be blamelessly employed, without any reflective appreciation of its epistemic status, just in case inferring according to P is a precondition for having the concepts ingredient in it." Boghossian talks of a blind, but blameless (entitling) inference. This mixes psychological characteristics (blindness, non reflectivity) with a justificatory one (blamelessly). But my problem is with the former. Boghossian's (2002) formulation is: "If fundamental inferential dispositions fix what we mean by ours words, then we are entitled to act on these dispositions prior to and independently of an explicit justification for them." But the for the purpose (which is not Boghossian's, but mine) of spelling out what the dispositions are – which is certainly part of the answer to the psychological problem of how we can causally be moved to infer.- it presupposes an answer to this problem.

[38] See in particular "What is a theory of meaning? (II) , "What do I know when I know a language?" in Dummett 1993, Dummett 1991, ch. 4

account, then, what is missing in the Tortoise is the appropriate recognition of the assertability conditions of sentences of the *if P then Q* form.

The problem is that it is hard not to assimilate this view to what I have called above a form of reflective internalism. Even though the kind of knowledge that we manifest when we make elementary inferences such as A-Z is implicit, and when manifested not "in the forefront of our consciousness", it seems clear that what Dummett requires of knowledge of meaning is that such knowledge be, in Williamson's phrase, "luminous". What one must know in order to know that a certain connective has a certain assertion-conditions, is that one *knows that one knows it*, or at least one must be in position to know that one knows it. (Williamson 2000:111). My point here is not, unlike Williamson, to argue that the thesis that knowledge is "luminous" is false, *i.e* to argue against the thesis that to know that P one must know that one knows that P. It is just that if one accepts this thesis, or even the weaker one that to believe one must believe that one believes, or that one has a reflective awareness of what one believes, it is not clear that we move further than reflective internalism. The way in which Dummett describes our awareness of meaning also suggests that he might be close to what I have called above non reflective internalism. But I doubt that he would accept here the notion of rational intuition which this view presupposes. So it is not obvious that the Tortoise's challenge about the force of logical reasons has been appropriately answered. In any case, an externalist reading of the relationship between our logical reasons and our particular inferences seems to be required. [39]

It is not the place here to attempt to describe what a possible solution to the problem of the force of logical reasons could be and I would not pretend to have one. I agree, however, with Dummett that one must give an account of our implicit knowledge of rules, and of the dispositions which are associated with their use. I also agree with him that there must be some sort of explanation of how these dispositions are manifested, and how they can be prevented from being manifested in some cases. But I disagree with him that we must impose strong internalist requirements about our knowledge of rules, and I am less confident that the explanation in question can proceed at the level of the description of a practice where everything is manifest and open to view, and that it can be

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[39] As Olav Gjelsvik pointed out to me, suggesting that at this point the epistemology of logical knowledge might benefit from virtue epistemology. In discussion Peter Pagin has also suggested to me a very interesting way to relate interpretation (1) and (4) of the paradox. We always have to formulate rules of inference so that they do not become other propositions. So we formulate them in a metalanguage. Rules in this sense are always general, whereas their applications are always particular. One has always to have variables. Hence there is always a gap. This gap is related to the causal gap between normative reason and motivation reason. I have given some hints of a solution in Engel 2001 and *to appear*. In Engel 1994, I argued that some notion of tacit knowledge of a theory of meaning is legitimate.

completely free from causal considerations. Unless we answer these questions the Tortoise's challenge might well be still with us. [*]

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