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Are shadows transparent? An investigation on white, shadows and transparency in pictures

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Abstract : Shadow perception and transparency perception appear to use very similar rules, to the point that from the perceptual point of view shadows have been considered an instance of transparent objects. We claim that in spite of the similarities, shadows ought not to be considered transparent entities. The discussion has consequences both for the issue of the location of shadows, and for the conceptualization of transparency, and it provides insight into the inner complexities of the conceptual structure of visual concepts.

Keywords: shadow perception; transparency perception; pictorial representation

“No black, no white is transparent”/“Nessun bianco o nero è trasparente” (Leonardo da Vinci, *Scritti Scelti*, ed. M. Brizio, Turin 1962, p. 417, f. 23 verso)

An accurate description of how cast shadows look like is a relatively difficult phenomenological endeavor. Cast shadows have a shape – although oftentimes a blurred one; but do they have a color, and if so, what is their color? The same could be asked for their location. Are they in the surface they are cast upon, or are they somewhat above it, in the way a thin superposed film-like layer could be? Looking for a method for answering these at first sight exceedingly marginal questions can be useful in order to investigate some of the preferences and operations of the conceptual system. We would then be able to move to more general questions, such as what is for the concept of a physical entity to leave the possibility open that the entity’s location is only partly specified. Given the intrinsically visual nature of shadows, this inquiry has to take into account some constraints derived from the architecture of the visual system. In this paper I’ll pursue the issue of the location of shadows by testing it against the visual phenomenon of transparency, for two reasons: rules for shadow perception as close kins of rules for transparency perception, and transparency imposes three-dimensional ordering: if shadows are transparent, then their location would

be specified unambiguously. Questions to be discussed here are thus whether shadows are transparent; whether they have three-dimensional ordering; or whether white can be transparent.

The visual nature of shadows

An object's reflectance measures the proportion of incident light that is reflected by the object's surface. An object's luminance measures the light reflected by the object that travels to and is measured by the eyes of the observer. Shadows affect illumination, that is, the quantity of light that reaches the object, and hence affect luminance, which is simply the product of the illumination by the reflectance.

Shadows are physical entities, but among physical entities they are peculiar, insofar as they are not material. They are located in space without occupying the space they are located at. Above all, shadows have intriguing visual properties. The concept of a shadow is partly a visual, and partly a causal concept (shadows are the *result* of a *blocking action* by an *obtruder* on incident light). There are visual shapes and tactile shapes; a square can be such for the sight and for the touch; but there are only visual shadows (bar some inconsequent metaphors). Furthermore, the concept of a shadow is the concept of something with a visual boundary (Casati 2000). Even if the physical conditions for casting a shadow are present, in the absence of a visible boundary one is not prepared to accept that a shadow is there. The night is the shadow of the Earth, but it is not conceptualized, let alone seen as a shadow, precisely as it lacks a visual boundary, except perhaps when a Moon eclipse is observed. And indeed, the visual system deploys quite a lot of ingenuity in chasing after shadow boundaries, in particular in distinguishing boundaries of reflectance, such as the ones created at the limit of a green leaf on a white stone, from boundaries of illumination such as the ones created by a shadow on the white stone. Mistaking the latter for the former would entail taking the ephemeral for the permanent; mistaking the former for the latter would entail the opposite mistake. Both mistakes can have unwelcome consequences in ecological situations. The ephemeral is the mark of a moving object; the permanent is a clue to classification via properties of surfaces. Somewhat anecdotally, picture yourself as perceiving a portion of the visually striped skin of an animal: is it a zebra or is it a lion instead, in the scattered shadow of a palm-tree?

The visual system is somewhat able to discount the effect of shadows and to deliver a measure of the invariant property of reflectance. This has the consequence that a shadow does not change the perceived color of an object: a white stone upon which is cast the shadow of a leaf appears uniformly white, in spite of the very large difference in luminance between the lit up area and the shaded area.

Suppose that you can see the shadows' boundaries, thereby noticing that a shadow is present. What are the features of these boundaries that make you perceive a shadow? Not all boundaries will do. It was long thought that blurred boundaries are necessary (Hering 1878): they aren't, although they help, and in spite of the fact that most real shadow boundaries are blurred. However, as noticed by Leonardo already, tracing a shadow's outline destroys shadow character. The important fact is that certain luminance relations holding between the areas on the opposite sides of the boundary must be respected whenever the illumination boundary crosses a reflectance boundary. This brings us to transparency, as in transparency too the interplay between luminance relations across a boundary are crucial.

Varieties of transparency and pictorial (cyclopean) transparency

In the environment there is transparency when some object is seen *through* and *behind* another object. Being a spatial property related to how *distant* are *two* objects from an observer, transparency is in most cases assessed binocularly. A transparent layer can be seen over a background; the background is seen through the transparent layer. You see the silver boundary of a glass pane and you see that as you move, what is to be seen within the boundary changes. Not only glass (or transparent *stuff*, like a plastic film) can be seen as transparent in this sense. A canopy, a fence, a net, a piece of fabric, are sometimes usefully described as transparent, especially based on what you see when you move relative to them, so that the background behind them can be revealed piecemeal. However, binocular information and more in general three-dimensional stimuli are not necessary to convey the sense of transparency, as it was known to painters for a long time. Indeed, when looking at a painting you do not use information that need to be retrieved binocularly. Conversely, transparent objects may not be sufficient to visually convey a sense of transparency. A piece of green transparent film on a white cardboard can be seen as an opaque green patch. Finally, in a painting you have to use an opaque stimulus in order to convey a sense of transparency.

In a classical paper, Metelli (1974) showed how the impression of transparency is produced whenever a visual mosaic obeys certain configuration and lightness rules. Importantly, the mosaic is totally two-dimensional, a juxtaposition of opaque patches on a surface. Call this specific, cyclopean sense of transparency *pictorial transparency*. Note first that the configuration aspects of transparency require the presence of x-junctions, that is, regions of the image where lines cross. This is why the green film placed at the center of the white cardboard fails to be seen as transparent – no x-junctions are involved. The surface that is seen as transparent must further be seen as unitary in color, and this can happen in some conditions. Consider three mosaics as in the following figure.

Although these are just mosaics, that is, juxtapositions of opaque tiles, a sense of transparency is conveyed in the central region of each.

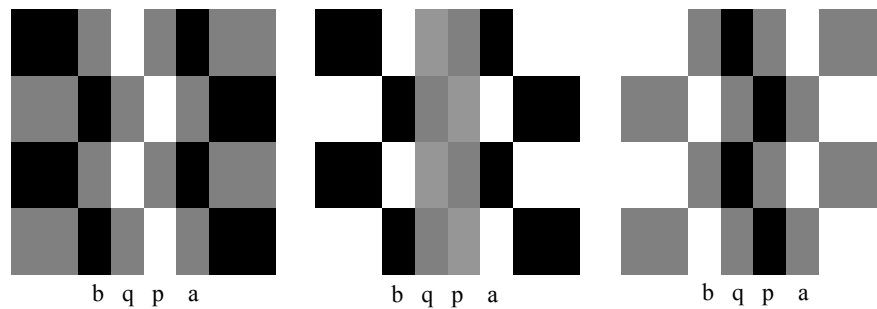


Fig 1. Some important predictions of the Metelli transparency Rules. Leftmost board: white stripe. Middle board: grey stripe. Rightmost board: dark grey stripe. The rules are explained in the text.

The Metelli rules for perceptual transparency predict the color and transparency value of the central band as a function of the relationship of the actual reflectance values of the tiles in the intermediate columns composing it (q and p) of each checkerboard with adjacent tiles (in columns b and a, respectively). If both values are lighter than those of the respective adjacent tiles, then the transparent band will be seen as having the lightest color in the scene (leftmost board). If they are both darker, then the transparent band has the darkest color (rightmost board). If one of the values of the central column is darker than that the adjacent tile, and the other is lighter, then the band will have an intermediate color (middle board).

Physical transparent filters act as in the rightmost board; they preserve the contrast of the tiles in the central area. Other cases of filters correspond to the leftmost and central board (an instance would be opaline glass, for instance). However, the central area appears, in those cases, less transparent than the one in the rightmost board.

The Metelli rules give values for color attribution once order is specified (given that figural conditions are respected). However, three-dimensional order is not specified. In the above figure we see that the transparent band is *above* the board; but how do we get this impression? Why don't we see a transparent checkerboard above an opaque band? The Metelli rules are unable to provide an answer because they work only "horizontally" as if it were. The Adelson Anandan Anderson (AAA) rules provide order by examining what happens at an x-junction. Kitaoka (2005) nicely summarizes them graphically in the following way.

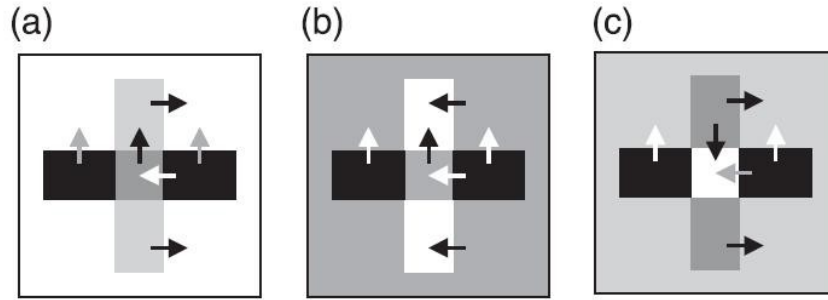


Fig. 2 The AAA ordering rules according to Kitaoka (2005).

The arrows flow from the darker to the lighter of two adjacent surfaces. In (a) the three-dimensional order is unique: we see a transparent grey band over a darker band but we cannot see a transparent dark band over a light grey band. When crossing the x-junction (consider the four tiles in the top right corner) contrast polarity is preserved along the horizontal (both tiles are darker than their adjacent tiles), but is inverted along the vertical boundary (one tile is darker, the other is lighter than its adjacent tile). In (b) order is not specified; we may as well perceive a white transparent band over a black band and conversely. Contrast polarity is preserved along both boundaries across the x-junction. In (c) no transparency is seen; contrast polarity changes along both boundaries.¹

From the above analysis it follows that the key feature of visual transparency is that one and the same area in the visual mosaic is labeled with two different properties, i.e. two different shades of color; which are then seen as separated in space (as one band is seen above the other – although it may not be specified *which* one). The color of the relevant tile of the mosaic is such that part of it can be attributed to the underlying band and part to the overlaid band.

For this ‘color split’ to occur, it is *prima facie* – based on the idea that the transparent layer behaves like a filter – requested that the overlaid band darkens the bottom band as much as it darkens the background. However, as we have seen, other conditions apply that block the possibility to consider the impression of transparency as the simple superposition of filters (i.e., filters whose combined action is a form of subtractive color synthesis).

In particular, see-through patterns are produced also when the differences in the background contrasts, albeit preserved, are attenuated, as

¹ Kitaoka 2005 further enriches the issue by distinguishing object and layer transparency; for instance in the leftmost Metelli checkerboard one may see two dark transparent vertical stripes over a light checkerboard. We shall not delve into this latter interpretation here.

in the two left patterns in fig. 1. These configurations correspond to ecological situations in which it is possible to see through semi-opaque, translucent materials, such as alabaster; or through a white, finely woven fabric, such as the one some stockings are made of. At this point we turn to the issue of what it is for a *white* surface to be visually transparent.

Can white be transparent?

Wittgenstein (1977 3: 189) argued that different color concepts entertain different relations with the notion of transparency. In particular, he claimed that whereas it is possible that red be transparent, there cannot be transparent white. In Casati 1989 I argued that an explanation of the alleged incompatibility of whiteness and transparency is available on purely phenomenological grounds. Whiteness and transparency would be incompatible because white does not support degrees of brightness. If the tiled area – the part of the mosaic which is the vehicle for transparency – is to replicate the see-through contrast, then it must be such as to allow for degrees of brightness. This is possible for all colors that are between white and black (there is a dark red and a light red, a dark grey and a light grey), but not for white and black itself: white and black are the extreme points of the brightness spectrum, and as points they have no internal complexity. In the light of the discussion of the Metelli rules above it is possible to reconsider that suggestion by making the hypothesis that *two* notions of transparency are at work, *clear* transparency (the one that uniformly dims the contrast in the background) and the somewhat oxymoron *opaque* transparency (the one that uniformly lightens the contrast in the background). Transparent white would then be possible only in the opaque sense of transparency.

However, it is also possible to simply dismiss the suggestion by saying that in order for something to appear white, it needs not be uniform white or be presented by a uniformly white vehicle. The depiction of a wrapped handkerchief may well be rendered through a vast array of grey spots. Hence if white cannot be transparent, it is not because it cannot be the vehicle of transparency simpliciter (it can) but because we want it to be the vehicle of transparency in a very specific sense, the sense in which all components of the vehicles, taken individually, are white. In a mosaic conception of perception, one may want to say, all tiles of a white object are taken to be individually white. But there are reasons to deny a mosaic view.

Visual transparency and the transparency of shadows

There are two important consequences of the perceptual account of visual transparency for the question of the visual transparency of shadows.

First, if shadows are to be seen as transparent, and if transparency is conveyed by x-junctions, then shadows they have to straddle luminance boundaries. The only shadows, if any, that can be seen as transparent are the ones which are cast on tiles, surface discontinuities, etc.

Second, the fact that the impression of transparency is conveyed by conditions in which light is not only subtracted, but also added, marks a distinction with the ecological, normal case of shadows. Indeed, the shaded area is always dimmed relative to the surrounding. An immediate consequence of this is that the only type of transparent material that could be used to imitate a shadow is a dark film. Research in visual perception appears to confirm this (Cavanagh, Leclerc 1989). Negatives of pictures, in which shadows and shaded areas are made lighter than their surroundings, are hard or impossible to parse – for instance, face recognition is severely impaired if it is to rely on ‘reversed’ shading – suggesting that ‘whitened’ shadows are not seen as such and retrieved for reconstructing shape, let alone being recognized as shadows.²

Finally, shadows cannot be represented by line drawings (Kennedy 1974). Lines in drawings signal borders of occlusion or else discontinuities that may become occluding borders if the object were seen from a different angle. Luminance boundaries are neither, and if a shadow is depicted by depicting its outline not only the shadow is not recognized, the remainder of the scene is also made difficult to parse.

To what extent is shadow perception an instance of transparency perception?

As we have seen, the key feature of transparency is that one and the same visual region ‘stands for’ two different surfaces, the overlaid transparent surface and the underlying surface (limiting the case to two surfaces; a welcome experiment should ascertain how many distinct surfaces can be seen as superposed transparently). This entails that the color of the visually shaded area is seen as the consequence of the interaction of two distinct colors, those of the two surfaces. In other terms, the luminance value of the area is ‘split’ into two inferred reflectance values. The spatial location of the visually shared area is also resolved with the attribution of two different distances from the observer: the transparent surface – in the case of unambiguous ordering – is seen above the background surface (in the bistable ordering one of the two surfaces is indifferently seen above or

² However, the stimuli for the Cavanagh experiments are mainly shadows that do *not* straddle reflectance boundaries. A welcome experiment could determine whether stimuli in which the non-subtractive conditions for transparency are associated to a candidate area for a shadow allow shadow recognition, thereby trumping the lightness-dimming condition of a shadow. For instance, a false shadow – cast on a checkerboard – which is transparent and lighter than the surrounding.

behind the other). Let us consider how these two factors (color and location) could transpose in the perception of shadows.

Shadow can only conform to the Metelli rule that correspond to the clear filter (the rightmost checkerboard in fig. 1, which can indeed be seen both as a dark superposed film-like layer, and as a shadow cast on the board).³ Symmetrically, a spot of light will conform to the rule for transparency in the leftmost checkerboard in fig..., which can indeed be seen both as a white superposed layer, and as an intense spot of light on a black-and-grey checkerboard. Neither a shadow interpretation nor a spot-of-light interpretation is however available for the middle checkerboard. The Metelli rules can account for these three phenomena in a simple way. First, the dark filter/shadow interpretation in the rightmost board is grounded in the fact that both tiles q and p are darker than their neighbors b and a respectively. Symmetrically, the spot of light/white transparent layer interpretation in the leftmost board is grounded in the fact that both tiles q and p are lighter than their neighbors b and a respectively. However, no lightspot and no shadow can behave in such a way as to produce the results of the middle checkerboard: for here the central tile q is *lighter* than its neighbor b, whereas central tile p is *darker* than its neighbor a. The particular distribution of luminances of tiles cannot be explained as the result of a single agent (a shadow, or a spotlight) because shadows cannot both darken and lighten (they can only darken) and spotlights cannot either (they can only lighten).

Compliance with the relevant Metelli rule is only a necessary condition for perceiving a shadow; other elements should be factored in for delivering the shadow appearance.

Arnheim (1974) argued that the phenomenology of cast shadows perception is an instance of transparency.

“I look at the small wooden barrel on the shelf. Its cylindrical surface displays a rich scale of brightness and colour values... But this description is correct only as long as I examine the surface inch by inch, or better, if I scan it through a small hole in a piece of paper. When I look at the barrel more freely and naturally, the result is quite different. Now the whole object looks uniformly brown. On the one side it is

³ This is another sense in which shadows can be said to be transparent: they are not *opaque*; they act as *clear* filters.

overlaid with a film of darkness, which thins out and disappears while an even thicker layer of brightness begins to replace it. Over most of its surface the barrel shows a double value of brightness and colour, one belonging to the object itself and another, as it were, draped over it—a transparency effect. This happens even though the eye receives one unitary stimulation from each point of the object. Perceptually, the unit is split up into the two layers. Here is a phenomenon that requires a name. The bottom layer will be called the object brightness and object colour of the barrel. The top layer is the illumination. Just as in central perspective a system of convergence is imposed upon a setting of shapes, *illumination is the perceivable imposition of a light gradient upon the object brightness and object colours in the setting*. The superposition observed on the surface of illuminated things is... a transparency effect.” (p. 309-310).

The idea that shadows are a sort of transparent layer is apparently indirectly endorsed by Adelson and Anandan:

“Many physical phenomena can produce transparency. For example, dark filters, specular reflections, puffs of smoke, gauze curtains, and cast shadows, all combine with patterns behind them in a transparent manner.” (Adelson and Anandan 1990, p. 77).

However, the description grounding Arnheim’s idea that shadows are transparent can be questioned. Transparency requires (at least) the impression of *two* (or many such) surfaces’ being seen, a background surface and a superposed surface (or many such). Illumination differs from transparency because there is only a background (object) surface to be seen, even though painters actually can reproduce illumination by superposition of transparent layers, as Arnheim correctly points out. In the case of a cast shadow, the separation in space is not available to account for the luminance split. Capitalizing on the discussion so far: in order to perceive a certain area as a shadow, x-junctions are not requested as they are in the case of transparency. (At the same time, a lone shadow, one which cannot be easily traced back to an obtruder casting it, may be hard to perceive as a shadow – it is in this case that blurred boundaries can help.). This means that shadows can be seen as shadows even though they do not straddle a luminance boundary; but straddling a boundary is requested for a transparent surface to be perceived. Hence the general case of shadows’ being transparent is not viable. The door is still open to evaluate whether shadows that do straddle a boundary are perceived as transparent. But a general case cannot be made that a shadow float above, or is not in touch with, the surface it is cast upon. It is not a film-like area superposed to the object. The separation between background surface and transparent layer that exists in the case of transparency, if it is to exist in the shadow case, must of a different kind.

Space occupation is jealous, at least relative to some contexts. Two entities of the same kind – two apples, or more generally two material objects – cannot occupy the same spatial region at the same time. Events seem to arouse less jealousy: the rotation of the sphere and its warming up

at the same time are co-located (Davidson 1969). Auditory perception is tolerant towards co-location. Chords are heard as complex entities whose components are heard as co-located. It is also generally recognized that two entities of a different kind can occupy the same region of space: a statue and the portion of marble it is made of – if you agree on the fact that the statue is not, or is not just, the portion of marble; a hole and the screw in it, if you agree that the hole is a full-fledged spatial entity. A three-dimensional shadow can be made coincide with a hole. It may even be argued that a body coincides with the shadow created by its outer surface. However, the separation of shadow and surface is not to be thought along the model of the separation of spatially coincident shadow and hole; after all, it is even conceivable that two different shadows, artfully cast from two different objects, coincide in space.

To take stock: *Where* is then the cast shadow? On the face of it, it is not laid *over* the surface it is cast upon. But it is not spatially unrelated to the surface either (there is a sense in which it is ‘there’ – and it surely isn’t somewhere *else*). We can think of three lines of answer to the question. A radical, eliminativist solution to the issue is to consider that there are no shadows after all, only areas that are (more or less) shaded. A different, straightforward if counterintuitive resolution of the issue is to consider that the shadow literally is *in* the surface. This is tantamount to saying that there is a level of visual representation in which objects are represented as ideal volumes of space, provided with ideal two-dimensional surfaces, and that surfaces are penetrable and can spatially coincide with other surfaces. Finally, one can simply say that shadow location is specified only up to a point. The visual concept of a shadow indicates in two-dimensions where the shadow is on a surface (its superficial address, as if it were) without indicating its three-dimensional relationship to the surface.

None of the above revisionary accounts supports shadow transparency, as the latter requires the presence of two, spatially distinct layers. Eliminativism gets rid of the shadow altogether, thereby disposing of the bearer of transparency. The idealized volume view forces the shadow into the surface; and the indeterminate location account prevents it from being anywhere in three-dimension relative to the surface; either way, the shadow cannot be above the surface, as is requested by the conditions for transparency.

An intriguing complication is provided by partly coincident shadows, typically cast by two different objects and two different light sources. Consider again what would happen in the case of the superposition of transparent filters.

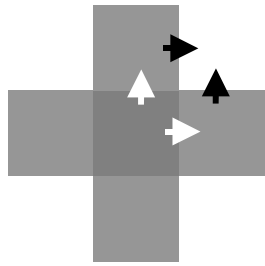


Fig. 3: Partly coincident shadows.

Polarity-wise, we are here in a case similar to that of fig. 2b, except that the shaded area is always dimmer than the background. Polarity is not changed across the boundary, and the result is bistable transparency: either strip can be seen ‘through’ the other. If we turn to shadows, we have three possibilities. In the eliminativist account, there are only areas that are shaded relative to one light source (such as the arms of the cross) or relative to two light sources (such as the central area). The ideal object account forces both shadows into the surface of the object. The indeterminate location account deprives of content the issue of shadow location in the third dimension. Once more, none of the three accounts supports transparency; it is then to be ascertained, by empirical means, whether patterns such as the one in fig. 3 is perceived as a case of transparency when shadows are involved. In this respect it is important to observe that in the shadow case the x-junction concerns boundaries that exclusively depend upon illumination. The empirical hypothesis is then that whenever the visual system is able to establish that the x-junction concerns illumination boundaries (and no reflectance boundary is involved) no sense of transparency is conveyed by the image.

The above discussion points towards the following conclusion. The concept of transparency is multiply ambiguous, and has an internal complexity that requires a lot of careful inspection to unearth. So is, for close reasons, the concept of a shadow. Both concepts have a visual side and a causal side, and the interaction between the two sides become problematic when spatial features come into play.

I shall close my discussion by presenting two side issues.

Are shadows colored?

We asked whether shadows are transparent. A cognate question concerns the color of the shadow. Is it grey? Admittedly, some shadows are

darker than others. And shadows of non-colored light are not colored (colored shadows usually result from the concurrence of two differently colored lights, so that the shaded area receives only one type of light, which thus colors the shadow). According to Mizrahi (2002: 186ff.) a pictorial conception of shadows informs our response to the question about the color of a shadow. A shadow cast on a white surface is called ‘grey’ because grey is the color we would use to *depict* it. In itself, Mizrahi argues, this particular shadow is white as the surface is white; it would be yellow had the surface been yellow. However, the pictorial conception of shadows may not be available to the untutored. Not only that; we may not want to call ‘yellow’ a shadow cast on a yellow surface. Indeed, by so doing, we would neglect the shadow’s contribution to the luminance value of the surface. Instead, it looks as if the visual system attributes to the shadow a grey value that results from a calculation of the quantity of light that is prevented from reaching the surface, relative to a normalized white value. Thus, it is not at all unjustified to say that shadows are grey.

Making transparent and making visible

Our discussion of the relationships between shadows and transparency have relied on the “see-through” sense of transparency. In yet another sense, one may say that to be transparent is to make visible. Shadows can then be construed, in some circumstances, as transparency-makers. In bright sunlight it may be impossible to read a white sheet of paper covered with grey writing; the writing texture may however become visible only if a shadow is cast on the sheet (this fact was pointed out to me by J.M.Kennedy). The shadow here is like a transparent hole in an opaque surface, beyond which features of an underlying surface are revealed. In a sense, if white cannot be transparent, it turns out that black is the absolutely transparent color, the one that offers no resistance to the sight.

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