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# Can mereological sums serve as the semantic values of plurals?

David NICOLAS<sup>1</sup>

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## 1. Introduction

Schein (1993), Oliver and Smiley (2001), and Rayo (2002) have—inspired by Boolos (1984)—put forward a dilemma to show that characterizing the semantics of plural expressions in usual first order logic is unsatisfactory.

On the one hand, if we use sets to represent plurals, we run into the contradiction discovered by Russell in naive set theory. Indeed, the sentence *There are some sets such that a set is one of them just in case it is not a member of itself* seems comprehensible and true, yet it cannot be represented in a systematic semantics based on sets, for the resulting logical representation would be contradictory. If plural expressions were taken to denote sets, the sentence would indeed be represented as follows: ‘There is a set  $x$  such that, for any  $y$ ,  $y$  is a member of  $x$  just in case  $y$  is a set that does not belong to itself.’ But as known since Russell, there can be no such set.

On the other hand, if we use mereological sums, the semantics turns out to be too weak.<sup>2</sup> Indeed, there exist count nouns  $M$  and  $N$  such that an  $M$  is not an  $N$ , but the  $M$ s and the  $N$ s have the same sum. The semantics then attributes the same truth-value to all sentences of the form *The Ms Qed* and *The Ns Qed*, whatever the collective predicate  $Q$ . With certain predicates, however, these sentences have, intuitively, opposite truth-values.

These authors have concluded from this that stronger logics should be used. Accordingly, McKay (2006), Oliver and Smiley (2006), Rayo (2002, 2006) and Yi (2005, 2006) have studied the properties of “plural logic”, a logic that extends first-order logic by adding plural variables, plural quantifiers and plural predicates. Plural logic seems to have very attractive features. It has the expressive and deductive power of monadic second order logic. But while second-order logic is committed a vast ontology of concepts, plural logic is arguably committed to nothing more than the objects quantified over by the usual, singular, existential and universal quantifiers.

Establishing that plural quantification and predication are irreducible to singular quantification and predication is a crucial step in assuring the legitimacy of plural logic. So in this paper, we want to examine the second horn of the dilemma sketched above, which we find in need of re-evaluation. Friends of plural logic have argued that a semantics of plurals that uses mereological sums would be too weak, and they have adduced several examples in favor of their claim. However, they have not been systematic, and they have not considered various possible counter-arguments. With McKay, let us call *mereological singularism* the idea that mereological sums can serve as the semantic values of plurals. How strong are the arguments against this view?

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Acknowledgments to be added.

<sup>2</sup> As stressed by Rayo (2002), if mereological sums are used, the semantics is *not* necessarily paradoxical. See also the discussion of Schein (1993) in section §2.9 below.

After examining in a little more detail what mereological singularism is, we review and discuss the various arguments that have been presented against this position.<sup>3</sup> As we will see, several arguments are easily answered, while some others turn out to be more problematic.

## 2. Arguments against sums and replies

Let us first explain a bit more what mereological singularism is. Several researchers (e.g. Leonard & Goodman 1940, Massey 1976, and Link 1983, 1998) have proposed that *sums* should serve as the semantic values of plural expressions. What entities are these?

The modern definition of mereological sum (Simons 1987, Varzi 2006a) uses the notion of overlap, which is linked with that of part as follows. *a* overlaps *b* just in case something is part of *a* and part of *b*. Then, the mereological sum of *a* and *b* is the object *s* such that something overlaps *s* just in case it overlaps *a* or *b*. This sum may be noted *a + b*. In addition, a generalized notion of sum is defined for any monadic formula  $\phi$ . The mereological sum of anything that satisfies  $\phi$  is the object *s* such that something overlaps *s* just in case it overlaps something that satisfies  $\phi$ . It may be noted  $\sigma[x / \phi x]$ . In particular, for any count noun *N*, the mereological sum of the *Ns* is the object *s* such that something overlaps *s* just in case it overlaps an *N*.

The notion of part is taken to be a partial ordering. Moreover, it is supposed that when some objects have a mereological sum, this sum is unique. This is usually guaranteed by the so-called axiom of strong supplementation:

$$\forall x \forall y [ \neg (y \leq x) \rightarrow \exists z (z \leq y \wedge \neg Ozx) ]$$

This axiom makes the mereology extensional: any objects that have the same parts are identical. Finally, for any monadic formula  $\phi$ , an axiom schema guarantees that, whenever at least one thing satisfies  $\phi$ , there exists the mereological sum of everything that satisfies  $\phi$ .

The basic idea of mereological singularism (which may have to be refined) is then the following. Consider a predicate and the objects denoted by its argument. Let us call the predicate *collective* if it applies to these objects as a whole. Thus, in *John and Mary carried the piano together*, the predicate *to carry the piano together* applies to John and Mary as a whole. It does not apply individually to John, and individually to Mary. (By contrast, the predicate *to be tall* is not collective, but *distributive*. In *John and Mary are tall*, it only applies individually to John, and individually to Mary.) Mereological singularism then says that, for any collective predicate *Q*, a sentence like *John and Mary Qed* is true just in case the sum of John and Mary Qed. Likewise, for any count noun *N* and collective predicate *Q*, a sentence of the form *The Ns Qed* is true just in case the sum of everything that is an *N* satisfied *Q*. Under this conception, a collective predication is taken to be (reducible to) a singular predication: a predication in which a property is ascribed to one object, a certain sum.<sup>4</sup>

We now present and discuss various arguments against mereological singularism. We organize the discussion so that each argument presented is met by a reply of a new kind.

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<sup>3</sup> The opponents to mereological singularism certainly differ as to which arguments they find effective and would want to endorse.

<sup>4</sup> For the singularist, a distributive predication is equivalent to a conjunction of singular predications. Thus, *John and Mary are tall* is taken to be equivalent to: *John is tall and Mary is tall*.

## 2.1. Do abstract objects have sums? What about entities from different ontological realms?

According to Simons (1982a, 1982b), the notion of mereological sum has a clear meaning only in the case of concrete entities like material objects or events. Consider abstract things like, say, numbers. What could the mereological sum of several numbers be? Likewise, take entities from different ontological realms. What could the sum of a cat, a boundary, and a number be? Simons thinks that the notion of mereological sum does not apply in such cases. Still, this does not prevent us of talking of such entities together (and even counting them in certain contexts). So all this would show that plural reference, which is universally applicable, has nothing to do with mereology.<sup>5</sup>

However, one need not agree with Simons's conclusions. Yes, countenancing such mereological sums may feel odd. But this does not turn them into impossible entities. And after all, while we may have a comparable feeling with a set comprising a cat, a boundary and a number, we do not want to conclude from this that there is no such set. Mereology (just like set theory) is characterized by certain axioms. A priori, such axioms are subject-neutral. It is not necessary to limit their application to the realm of concrete and mono-categorical entities (cf. Lewis 1991).<sup>6</sup>

Uzquiano (2006) shows that certain combinations of assumptions concerning set theory and mereology are inconsistent. He concludes from this that mereology is not universally applicable: we should not think that it applies to sets, and so a conclusion similar to that of Simons would follow. However, the assumptions in question, notably concerning set theory, are not universally accepted. The inconsistencies Uzquiano points out can be avoided. For instance, Lewis (1991) eschews them by adopting a set theory with proper classes (where a proper class cannot be the member of a singleton set). Doing so may have a cost in the philosophy of mathematics, as Uzquiano thinks, but it is not clear why this should limit the options of someone who wants to develop a semantic theory of plurals in natural language.

## 2. 2. Plurals do not carry ontological commitment to mereological sums

Yi (1999, 2005) and McKay (2006, chapter 2) present a general argument against any singularist treatment of plurals. Let us illustrate it in the case of sums. Take a sentence like *John and Mary Qed*, where *Q* is a collective predicate, like *carried a table together*. For the mereological singularist, this is true if and only if the mereological sum of John and Mary Qed. Now, the right-hand side of this bi-conditional logically implies that there exists a mereological sum. According to Yi and McKay, this is an unwelcome consequence of the analysis, since the original English sentence, *John and Mary Qed*, does not logically imply the existence of a mereological sum.

The mereological singularist can answer as follows. The purpose of the bi-conditionals he proposes is to characterize inferential relations between various sentences of English. But of course, the metalanguage used (here, first order logic augmented with mereology) may bring its own commitments (in particular, to the existence of mereological sums), independently of those of sentences of natural language. Likewise, usual model theory carries a commitment to sets. A theorist proposing a model theory

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<sup>5</sup> This objection goes back to Frege (1884/1953)'s discussion of Mill's theory of numbers as properties of aggregates of things. According to Varzi (2006a, section §4.3), it was also prominent in early debates about the calculus of individuals and Goodman's nominalism.

<sup>6</sup> See also Varzi (2006b) for a defense of mereological universalism in a similar spirit.

for English is thereby committed to the existence of sets, in the same sense as a set theorist is. But he is committed to this solely because he uses sets as a tool, not because a simple sentence of English would logically imply the existence of sets.<sup>7</sup> The same is true when the theorist uses mereological sums.

This, however, raises a question. What is the relationship between a semantic theory of English and the tacit knowledge that English speakers have of their language? Some may be content to divorce their semantic theory from any claim about the tacit knowledge of English speakers. However, ideally, a semantic theory and a theory of tacit knowledge should go hand in hand. Therefore, it would seem a sound scientific attitude to draw an inference to the best available explanation in this case. If our best semantic theory of plurals uses mereological sums, then we should be ready to conclude that English speakers also make use of them, however unconsciously. But this does not modify what was noted above. Whether a theorist or an ordinary English speaker makes use of mereological sums to interpret a sentence of English, this changes nothing with respect to the fact that the sentence, by itself, does not logically imply the existence of sums.

McKay presents another, related argument. Whether mereological sums exist (and in what conditions) is a metaphysical question. The semantics of plurals should be independent of issues of this kind. Obviously, a similar point would apply to sets as used in a model theory of English. The answer in each case is that the theorist is using mathematical entities as tools to characterize the truth-conditions of English sentences. When doing so, he is committed to the existence of these entities in the same way that a mathematician is, when developing or using a theory of these entities. Mathematical practice suggests that such a commitment poses no problem, as long as the theory is consistent (see Linnebo forthcoming).

What precedes may also be seen in a somewhat different light (cf. McKay 2006). To analyze the semantics of English sentences containing plurals, the singularist uses a “substitute”, e.g. a set or a mereological sum. Therefore, his analyses and the representations he proposes are partly opaque, partly unfaithful. Indeed, his representations are about sets or sums, while the English sentences that are represented are, arguably, neither about sets nor about sums. It would be better to use a transparent medium of representation, such as plural logic.

But is transparency better, all other things being equal? In the present case, transparency means that plural quantification and predication are taken as unanalyzed primitives. By contrast, the mereological singularist maintains that plural quantification and predication can be analyzed in terms of well-understood theories, first order logic and mereology. Transparency might then appear as a loss.

So the evaluation has to be more global: how practical and fruitful is it to use this or that theory when characterizing the truth-conditions of English sentences? This question cannot receive an a priori answer.

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<sup>7</sup> Rayo (2007) makes a similar point.

### 2. 3. John and Mary met, but their mereological sum did not

Suppose John and Mary met. In English, it would be very odd to say the mereological sum of John and Mary met (McKay 2006). The same would be true if the predicate was replaced by *gathered*, *united*, *separated*, etc.<sup>8</sup>

Here is a first reply. The mereological singularist may want to develop a model theory, rather than a truth-theory. A characterization of truth-in-a-model specifies the truth conditions of sentences of the language relative to a model, i.e. relative to an arbitrary assignment of semantic values to the primitive vocabulary of the language. When doing so, the nature of the elements in the models that are used does not matter. Using these models, the theorist just wants to characterize the inferential relationships between various sentences of English.

In particular, we may build a model for a sentence like *John and Mary met*. We may take it that the sentence is true in the model just in case  $I(\textit{John and Mary})$  is in  $I(\textit{met})$ , where  $I$  is the interpretation function. Moreover, given our hypothesis (mereological sums are the semantic values of plurals), we have:  $I(\textit{John and Mary}) = I(\textit{John}) + I(\textit{Mary})$ , the sum of  $I(\textit{John})$  and  $I(\textit{Mary})$ . So the sentence is true in the model just in case  $I(\textit{John}) + I(\textit{Mary})$  is in  $I(\textit{met})$ .

So far, so good. But the mereological singularist can talk about mereological sums and make claims about them. So, what about the sentence *The sum of John and Mary met*? The original problem recurs if, in any model, the subject of this sentence is taken to denote  $I(\textit{John}) + I(\textit{Mary})$ .<sup>9</sup> Insofar as this is a natural requirement, going from a truth-theory to a model-theory does not appear to be really helpful for the mereological singularist.

The best option seems to be to bite the bullet. The theory says that *John and Mary met* is true just in case the sum of John and Mary met. As a naïve speaker of English, we are likely to find the right-hand side of this bi-conditional odd. But what is the source of the oddness felt? The singularist can claim that this is because we are using our intuitions about the normal uses of *sum* and *met*. Technical and mathematical terms like *sum* do not refer to animate things, while the predicate *met* demands that its subject be animate. The restrictions of selection of the predicate are thus not satisfied, hence the oddness.<sup>10</sup> Similarly, according to the Davidsonian analysis of action sentences, *Brutus stabbed Caesar violently with a knife* is true just in case there was an event that was a stabbing of Brutus by Caesar, that was violent, and that was with a knife (Pianesi & Varzi 2000). The right-hand side of this bi-conditional *is* odd, but in itself, this does not disqualify the analysis.<sup>11</sup>

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<sup>8</sup> See Oliver & Smiley (2001) for an analogous argument.

<sup>9</sup> Michael Potter pointed this out.

<sup>10</sup> The mereologist uses the term *sum* in a novel way. Given the axioms of mereology, there is no reason why the sum of animate things should be inanimate. But our linguistic intuitions do not change so quickly.

<sup>11</sup> This line of response was suggested independently by Pascal Amsili, Brendan Gillon, and Øystein Linnebo.

#### 2. 4. Two men wrote a book, but the sum of their molecules did not

Oliver and Smiley (2001) invite us to consider two men and their molecules. Suppose the men have written a book together. Then their molecules have not, though the sum of the men is identical with the sum of their molecules.

The mereological singularist can reply as follows.<sup>12</sup> As indicated at the beginning of section §2, the notion of mereological sum depends on those of part and overlap. In English, we can say that a molecule is part of a man, and that a man is part of two men. The same expression *is part of* is used in both cases. But does it pick the same relation in both cases? Following Leonard and Goodman (1940), Oliver and Smiley suppose that it does and that this relation is extensional, and they conclude from this that the mereological sum of the two men is identical with the mereological sum of their molecules. However, common sense tells us that the two sums are distinct: the latter survives the spatial dispersion of the molecules, while the former does not since the men do not continue to exist when their molecules are scattered.

To be coherent, the mereological singularist therefore needs to give up (strict) extensionality. One way of doing so is to use at least two distinct notions of part: one that relates, e.g., a molecule to a man, and one that relates, e.g., a man to several men. (One could also try to use weaker axioms, perhaps along the lines of Bittner & Donnelly 2007).

Link (1983, 1998) uses two relations, one of individual part operating on a domain of individuals (noted ' $\leq_i$ '), and one of material part operating on a domain of matter (noted ' $\leq_m$ '). The domain of individuals is closed under an operation of individual sum (noted '+<sub>i</sub>') corresponding to the relation of individual part. (An ordinary individual, like John or Mary, is then an atomic individual: an individual that has no proper individual part.) The domain of matter is closed under an operation of material sum (noted '+<sub>m</sub>') corresponding to the relation of material part. A function associates to each individual its matter. The matter of the molecule is a material part of the matter of the man. However, neither the molecule nor its matter is an individual part of the man. On the other hand, if there are two men, then any of them is an individual part of their individual sum. Moreover, the matter of any man is a material part of the matter of the individual sum of the men. This framework, though clearly not economical, is meant to reflect our common sense and linguistic ontology.<sup>13</sup>

*Nota bene:* in what follows, we continue our discussion of objections to mereological singularism. Whenever possible, we identify lines of defense that are independent of Link's theory. There is indeed no reason to equate mereological singularism and Link's particular framework, and it is interesting to see which answers are independent from one another.

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<sup>12</sup> Alternatively, he might want to say that the sentence *The molecules of the men wrote a book* is true, but odd, for similar reasons than *The sum of the men met* is odd.

<sup>13</sup> Simons (1987) argues at length in favor of a partly similar system, where three notions of parts, and three corresponding notions of sums, are used. In particular, Simons uses a notion of class, according to which a man may be part of a class of men. However, in this system, the term "class" seems to be essentially a linguistic commodity used by the theorist: in reality, there are no classes; there are only individuals that can be quantified over plurally.

## 2. 5. The piles of grains are scattered, though the grains need not be

Rayo (2002) considers situations in which there are some piles of grains, and the piles of grains are scattered. According to him, in some of these situations, we may well judge the sentence *The grains are scattered* false.

Several answers may be attempted. One is to claim that the sentence *The Ns are scattered* can in general be analyzed as: the distance between any two Ns is greater than a contextually fixed distance. The truth of *The grains are scattered* in a situation would then entail that there is no pile of grain present in this situation. However, this purported analysis of the meaning of *scattered* seems to require too much and so to be incorrect.

Another possibility is, like in the case of men and their molecules, to maintain that a pile of grain is distinct from the sum of the grains that make it up. Indeed, the sum, but not the pile, survives the dispersion of the grains. Rayo (p.c.) counters this last argument as follows. Take the sentence: *The mereological sums, each of which is the sum of the grains making up a pile, are scattered*. In the circumstance imagined, it would be true, while *The grains are scattered* would be false. But the sum of the mereological sums of grains is identical with the mereological sum of all the grains, so the semantics must attribute the same truth-value to both sentences. (Here, some might want to deny that we have intuitions concerning whether the mereological sums are scattered. But we follow Rayo on this point: it seems to us that we do have intuitions about the sentence just above.)

Yet another possibility is to question the correctness of Rayo's original intuitions. Imagine a situation in which there are several piles of grains, and sentence (1) *The piles of grains are scattered* is true. What about sentence (2) *The grains are scattered*? Is it true or false? Or can it be either?

In fact, this depends on how sentence (2) is construed. If it is construed collectively, as saying of the grains, as a whole, that they are scattered, then it seems possible to assent to the truth of (2) whenever (1) is true. However, (2) can also be understood differently, in a manner that warrants the following reply:

(3) *No, the grains are not scattered. They are neatly arranged in piles.*

This construal corresponds to a partition of the grains. It construes sentence (2) as saying, of the grains in each pile, that they are scattered, and this is false in a circumstance in which sentence (1) is true.

So this example points to a phenomenon that is often noted in linguistics (see for instance Gillon 1992, Schwarzschild 1996).<sup>14</sup> This is the fact that sentences containing plurals are liable to collective, distributive, and intermediate construals (modulo the meaning of the lexical items, knowledge of the world and context of speech). To account for this, Gillon and Schwarzschild suppose that the interpretation of a plural sentence depends on the choice of a cover of the objects denoted by the noun phrase, where a cover is a set of sets whose union is the set of objects denoted by the plural expression. As a striking example, consider *Hammerstein, Rodgers and Hart wrote musicals*. This sentence is historically true, since Hammerstein and Rodgers wrote musicals together, and Rodgers and Hart did to. It is true neither on a collective construal nor on a distributive one, but it is true on an intermediate construal.

In the case of sentence (1), the predicate *scattered*, as applied to the subject *the grains*, favors construals that are not distributive (cf. the oddness of *The grains are each*

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<sup>14</sup> Though quite popular, the analysis of plural sentences offered by Gillon and Schwarzschild remains controversial. See notably Lasersohn (1995) for opposition.



*scattered*). But it unproblematically allows for collective and intermediate construals of sentence (2). The intermediate construal corresponding to a partition of the grains in piles makes (2) false. What about the collective construal of (2), i.e. *The grains, as a whole, are scattered*? Whenever (1) is true, it does seem possible to assent to the truth of (2) construed collectively.

Remark: above, we have made a simplifying assumption. We have indeed supposed that (1) is understood collectively. Suppose that it is understood distributively instead. Then (1) means: *The piles of grains are each scattered*. And then, sentence (2) is also true.

So, we must find a way to factor the construals of sentences (1) and (2) out. We suggest something like the following. Suppose (1) is construed relative to a certain cover. Then this cover must be the basis for the construal of (2). For instance:

- if (1) is construed collectively, then so is (2);
- if (1) receives a distributive construal, then (2) is construed as meaning: *The grains of each pile are scattered*.

When we do so, we see that sentences (1) and (2) are not problematic anymore. Whenever their construals correspond to one another, the sentences have the same truth-conditions.

## 2. 6. The non-atoms are not the atoms

In Link's framework, the domain of individuals is closed under the operation of individual sum.<sup>15</sup> An ordinary individual, like John or Mary, is an atom for the relation of individual part. *Atoms* (any individual that has no proper individual part) may thus be distinguished from *non-atoms* (any individual that has proper individual parts). Then, in any model that contains at least two atoms, the sentence *The non-atoms are not the atoms* is true. However, the individual sum of the non-atoms is identical with the individual sum of the atoms. Therefore, Link's semantics should predict that the sentence is false (cf. Schein 1993, chapter 2).

The mereological singularist can answer as follows. We grasp what it means for one thing to be identical with one thing. We can also understand a sentence like *The Ms are identical with the Ns*, where the predicate *to be identical* is flanked by two plural expressions. The interpretation of a sentence containing a definite plural noun phrase depends on the choice of a cover of the denotation of the noun phrase. When a sentence contains two definite plural noun phrases, two covers must be chosen and this gives rise to more possibilities of interpretation. For the above sentence, one construal makes sense, where each plural noun phrase is understood distributively. Under this construal, the sentence is true if each M is identical with an N, and each N is identical with an M. A priori, one could also imagine that each noun phrase is understood collectively (or intermediately). The sentence would be true if the Ms, considered together, were collectively identical with the Ns, considered together. However, what would this claim of collective identity between the Ms and the Ns mean? We have found no plausible answer to this question. Therefore, we take it that a sentence like *The Ms are identical with the Ns* can only receive a distributive interpretation. (The same holds for a sentence like *The Ms are identical to the N*. A distributive construal makes sense, which asserts that the Ms are each identical to the N. But we do not know what a collective construal would mean, which would claim that the Ms, taken together, are collectively identical

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<sup>15</sup> There is also a domain of matter, closed under the operation of material sum. It contains the matter of each individual and the material sums of quantities of matter.

with one thing, the N. What about an army and its soldiers? Can we not say that the army is its soldiers, and that the soldiers are the army? Yes, we can, but it does not seem that, doing so, we are really claiming literal identity between the army and its soldiers. After all, the army continues to exist when one of its soldiers die. Therefore, the army cannot be identical with any number of soldiers.<sup>16</sup>) Under the distributive interpretation, *The Ms are identical with the Ns* is true just in case each M is identical to an N, and each N is identical to an M.<sup>17</sup>

## 2. 7. Regions a and b overlap, but a minus b and b do not

Suppose we are talking about regions of space, and regions a and b overlap. Define a minus b as the object such that something x is part of it just in case x is part of a and x does not overlap b. Then, by definition, a minus b and b do not overlap.<sup>18</sup> However, the mereological sum of a and b is identical with the sum of a minus b and b. So mereological singularism seems to predict that *a and b overlap* and *a minus b and b overlap* should have the same truth-value (cf. McKay 2006, who uses a slightly more complex example).

Can this conclusion be avoided? Here is a first attempt, using Link's notions of individual part and individual sum. The idea is that *and*, when conjoining noun phrases, picks the operation of individual sum:  $+_i$  (the associated notion of individual part being  $\leq_i$ ). Minus picks an operation defined with respect to a different relation of part,  $\leq$ , which applies notably to space.<sup>19</sup> In a model theory, we then have the following (where I is the interpretation function):

$$I(a \text{ and } b) = I(a) +_i I(b)$$

$$I(a \text{ minus } b \text{ and } b) = I(a \text{ minus } b) +_i I(b)$$

We suppose that  $I(a \text{ minus } b) = I(a) - I(b)$ , where  $-$  is the minus operation defined with respect to  $\leq$ . So:

$$I(a \text{ minus } b \text{ and } b) = [I(a) - I(b)] +_i I(b)$$

Nothing in our axioms or in our subject matter can lead us to conclude that:

$$[I(a) - I(b)] +_i I(b) = I(a) +_i I(b).$$

However, the problem thus solved reappears if we make two apparently plausible assumptions:

- we can meaningfully use the notion of individual part and associated notions of overlap and minus in the object language;
- $I(a -_i b) = I(a) -_i I(b)$ .

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<sup>16</sup> Remark: someone holding that it is literally true that the soldiers are collectively identical with the army would have to hold the same concerning the non-atoms and the atoms. Namely, that it is literally true that the non-atoms are collectively identical with the atoms. The purported problem with the sentence *The non-atoms are not the atoms* would then disappear.

<sup>17</sup> Formally, the condition just given is:

$$\forall x (Mx \rightarrow \exists y (Ny \wedge y = x)) \wedge \forall y (Ny \rightarrow \exists x (Mx \wedge x = y))$$

More generally, consider a sentence of the form *The Ms Q the Ns*, where the verb is interpreted distributively with respect to both arguments. The sentence is true just in case:

$$\forall x (Mx \rightarrow \exists y (Ny \wedge Q(x,y))) \wedge \forall y (Ny \rightarrow \exists x (Mx \wedge Q(x,y)))$$

<sup>18</sup> The expression *a minus b* is a singular term. A proper name could have been used instead. We could have said: 'Let c be the object such that something x is part of c just in case x is part of a and x does not overlap b.' We just find it more perspicuous to use the term *a minus b* in the entire discussion.

<sup>19</sup> Øystein Linnebo and Alex Oliver both suggested this response.

For consider then two individual sums,  $a$  and  $b$ , which overlap (i.e. have an individual part in common);  $a -_i b$  exists, and  $[a -_i b] +_i b = a +_i b$ . This whole sentence can be seen as a sentence of the object language, which we understand, and which is true.<sup>20</sup>

Here is another possibility. Take two predicates of English,  $M$  and  $N$ . If their meanings bear no interesting relationship, then any interpretation of  $M$  and any interpretation of  $N$  should be possible and yield an admissible model. But what happens when the meanings of  $M$  and  $N$  are related? Dictionaries tell us that a bachelor is a man who has never been married. Hence, the theorist should consider only models in which  $I(\text{bachelor})$  is included in  $I(\text{man who has never married})$ . Similarly, the meanings of *overlap* and *is part of* are related. Thus,  $a$  and  $b$  overlap just in case there is something that is part of  $a$  and part of  $b$ . Therefore, when building a model for *a and b overlap*, the semanticist should only consider models that are also models of *There is something that is part of a and part of b*. Likewise, when building a model for *a minus b and b overlap*, the semanticist should only consider models that are also models of *There is something that is part of a minus b and part of b*. As a consequence, an admissible model of *a and b overlap* will never be an admissible model of *a minus b and b overlap*, even though the basic rules of the model theory say that both sentences are true in the same models.<sup>21</sup> While this suggestion is no doubt a bit puzzling, it is not clear exactly why.<sup>22</sup>

A more promising option is the following. It consists in claiming that *overlap*, as it appears in *a and b overlap*, is a covert reciprocal predicate. Now, take a covert reciprocal predicate, like *fight* in *John and Mary fight*. The semantic rules of English would indicate first, that this should be interpreted as: *John and Mary fight each other* (hence the qualification of ‘covert reciprocal’), and second, that the latter should be interpreted as *John fights Mary, and Mary fights John* (Langendoen & Magloire 2003). So in our case, the semantic rules of English would indicate that *a and b overlap* should be interpreted as: *a overlaps b, and b overlaps a*. This sentence contains no plural term, and so the alleged problem raised by McKay does not arise.<sup>23</sup>

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<sup>20</sup> See section §2.9 for a response to this objection that denies that we can meaningfully use the notions of Link’s “individual mereology” in the object language.

<sup>21</sup> The basic rules of the suggested model theory are as follows:

*a and b overlap* is true in model  $M$  just in case  $I(a \text{ and } b) \in I(\text{overlap})$

$I(a \text{ and } b) = I(a) + I(b)$

*a minus b and b overlap* is true in model  $M$  just in case  $I(a \text{ minus } b \text{ and } b) \in I(\text{overlap})$

$I(a \text{ minus } b \text{ and } b) = I(a \text{ minus } b) + I(b)$

Now, it is a plausible requirement that  $I(a \text{ minus } b) = I(a) - I(b)$ . Since  $[I(a) - I(b)] + I(b) = I(a) + I(b)$ , both sentences must have the same truth-value in any model.

<sup>22</sup> In the case of *bachelor* and *man who has never married*, a meaning postulate will say that *d is a bachelor* is true just in case *d is a man who has never married* is true, however  $d$  is interpreted. This postulate connects the interpretations of a syntactically simple predicate, *bachelor*, and those of a syntactically complex predicate, *man who has never married*. It will ensure that  $I(\text{bachelor}) = I(\text{man who has never married})$ . In the case of *overlap* and *is part of*, a meaning postulate will say that *a and b overlap* is true just in case *There is something that is part of a and part of b* is true, however  $a$  is interpreted and  $b$  is interpreted. The predicates *overlap* and *is part of* are syntactically simple, but the subject of *overlap* is syntactically plural. The effect of the postulate cannot be stated as simply as in the case of *bachelor* and *man who has never married*. But there is no reason a priori why this should be necessary. (On the use of meaning postulates to integrate formal and lexical semantics, see Partee (2005). She insists in particular on the fact that meaning postulates can put constraints on the relations that hold among the meanings of certain words without necessarily treating one word as more basic than another.)

<sup>23</sup> This line of response was suggested by Brendan Gillon.

## 2. 8. Counting

An argument often invoked has to do with counting (Oliver & Smiley 2001, Yi 2005, and McKay 2006). Suppose two men are present in a given circumstance. Then the following sentence is true: *The men are two*. However, given what we said so far, we should have:

*The men are two* is true if and only if the mereological sum of the men is two.

The right hand side of this bi-conditional seems to be false in the envisaged circumstance: one would want to say that the mereological sum of the men is one.

To counter this argument, it seems quite natural to follow Frege (1884), as Link (1998) and Wallace (manuscript) do. The idea is that counting requires the identification of a concept, or sortal, which specifies what is to count as one. What concept is appropriate may be provided by the sentence itself (its syntactic structure and the words it contains), or else by the context in which the sentence is uttered. Thus, the sentence *The men are two* is understood as *The men are two men*, while the sentence *The mereological sum of the men is one* is understood as *The mereological sum of the men is one mereological sum of the men*.

What about *John and Mary are two*? Here, the subject is not of the form *the Ns*, and so the concept appropriate for counting is not lexically expressed. This concept is identified in function of what is known about John and Mary. Typically, *John and Mary* will name persons, so the sentence will be understood as *John and Mary are two persons*.

What, then, of *These are two, while those are three*? In many cases, identifying the concept to be used for counting will be straightforward, e.g., SOLID MATERIAL OBJECT. Are there cases where no single concept would apply to the entities demonstrated by *these*? If what is demonstrated is meant to be perceptually manifest to speaker and hearer, then it seems that there must be such a concept, like SOLID MATERIAL OBJECT, or PERCEPTUALLY INDIVIDUATED FIGURE. If what is demonstrated is things mentioned in previous discourse, then *These are two* may be said while referring to a chair and to an idea previously mentioned. But the concept appropriate for counting would then be THING MENTIONED IN PREVIOUS DISCOURSE.

So, while this approach may face some difficulties,<sup>24</sup> it does have some plausibility. Let us now see how sentences like the above may be formally represented, using mereological sums and first order logic.

The sentence *The men are two* is understood as *The men are two men*.<sup>25</sup> It is true just in case:

$$\exists x_1 \exists x_2 [ x_1 \neq x_2 \wedge \forall y (\text{man}(y) \leftrightarrow y = x_1 \vee y = x_2) ]$$

More generally, *The Ms are n* (where *M* is a count noun and *n* a finite cardinal number) is true just in case there exist *n* things,  $x_1, \dots, x_n$ , which are pair-wise distinct, and which are such that, for any *y*,  $\text{My} \leftrightarrow \exists i (1 \leq i \leq n \wedge y = x_i)$ .

(*Nota bene*: we are not trying to define natural numbers. So we can use them in the analysis of adjectival predications of number. In the analysis just put forward, the whole sentence is a sentence of the metalanguage, which gives the truth-conditions of the object language sentence *The Ms are n*.<sup>26</sup>)

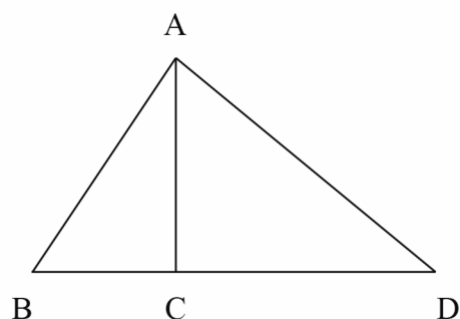
<sup>24</sup> See Grandy (2007) on the debates surrounding the notion of sortal.

<sup>25</sup> We are considering the reading of the sentence that means that exactly two men are present in the universe of discourse.

<sup>26</sup> Likewise, a sentence of the form *Some Ms are n* is true just in case  $\exists x_1 \dots \exists x_n$  such that:

$$\forall i \forall j (1 \leq i \leq n \wedge 1 \leq j \leq n \wedge i \neq j \rightarrow x_i \neq x_j \wedge Mx_i)$$

Let us consider here variations on an objection raised by Simons (1982a, 1982b). Suppose the count noun *triangle* is understood as meaning ‘area bounded by segments relating three points’. Imagine moreover that the universe of discourse is as depicted below, only containing points A, B, C, and D, and segments AB, BC, CD, DA, AC, and BD:<sup>27</sup>



Then, the sentence *The triangles are three* is true, since the triangles present in the universe of discourse are ABC, ACD and ABD. The truth of the sentence is predicted by the conditions given above.

But then, what about the following sentences:

*ABC and ACD are two triangles.*

*ABC, ACD and ABD are three triangles.*

Both sentences are true in the present context. But we get wrong results if we say that *ABC and ACD are n triangles* is true just in case the sum of ABC and ACD is the sum of n triangles and not more (where n is a natural number).<sup>28</sup> Indeed, the sum of ABC and ACD is identical with the sum of three triangles and not more (ABC, ACD and ABD). Therefore, the semantics predicts that *ABC and ACD are three triangles* is true, while *ABC and ACD are two triangles* is false.

*Nota bene:* if instead we said that *ABC and ACD are n triangles* is true just in case the sum of ABC and ACD is the sum of n triangles, then we would predict that both *ABC and ACD are two triangles* and *ABC and ACD are three triangles* are true, while the latter is intuitively false.

So can we do better? In such sentences, what is referred to is designated by proper names, so we could propose this. *ABC and ACD are two triangles* is true just in case ABC is a triangle, ACD is a triangle, and ABC is distinct from ACD. Could that be made to work generally?

Before answering this question, let us explore an alternative. The idea is simply to use Link’s individual sums. *ABC and ACD* is taken to refer to  $ABC +_i ACD$ , while *ABC, ACD and ABD* is taken to refer to  $ABC +_i ACD +_i ABD$ . Then it is true that  $ABC +_i ACD$  is the individual sum of two triangles and not more, while  $ABC +_i ACD +_i ABD$  is the individual sum of three triangles and not more.

However, we can consider a similar situation where what is counted are individual sums. Imagine that the universe of discourse contains just a and b and their individual sum c. In Link’s framework, a is an individual sum, and so is b.

<sup>27</sup> Simons used up to five overlapping squares. YuChun Chang suggested the simpler case with triangles.

<sup>28</sup> For any natural number n, the rules of the semantics should be able to predict whether a sentence like *ABC and ACD are n triangles* is true or false in a given context.

So both sentences below are true:

*a and b are two individual sums.*

*a, b and c are three individual sums.*

But  $a +_i b = a +_i b +_i c$ , so the theory just outlined predicts that *a and b are three individual sums* should be true, while it is intuitively false.<sup>29</sup>

We see that Link's move of using individual sums works well in most cases, except when we proceed to count individual sums themselves. The problem seems to be peculiar to conjunctions since, when the subject is simply a definite description, the semantics makes appropriate predictions. It correctly says that *The individual sums are two* is false, while *The individual sums are three* is true. So is it possible to give an appropriate treatment of conjunctions, independently of Link's move?

Consider a conjunction of the form  $NP_1, NP_2, \dots, \text{and } NP_k$ , where each  $NP_i$  is a singular proper name or a definite, count expression. The following may be proposed.

$NP_1, NP_2, \dots, \text{and } NP_k$  are  $n$  (altogether) is true just in case  $\exists x_1 \dots \exists x_n$  such that:

a)  $\forall i \forall j (1 \leq i \leq n \wedge 1 \leq j \leq n \wedge i \neq j \rightarrow x_i \neq x_j)$

b)  $\forall y [ NP_1(y) \vee \dots \vee NP_k(y) \leftrightarrow \exists i (1 \leq i \leq n \wedge y = x_i) ]$

In this formula, we are using the following conventions. When a noun phrase  $NP_i$  is:

- a singular proper name,  $NP_i(y) \leftrightarrow y$  is the object named by the proper name;
- a singular, definite, count expression,  $NP_i(y) \leftrightarrow y$  is the object denoted by the singular definite description;
- a plural, definite, count expression,  $NP_i(y) \leftrightarrow$  the predicate expressed by the singular count noun applies to  $y$  (i.e.  $Ny$ , where  $N$  is the singular noun).

These conditions do seem to deliver the right results. Are they ad hoc, or do they violate something like compositionality? We do not see why this should be so. The principle of compositionality can be stated as follows: 'The meaning of a complex expression is a function of the meaning of its parts and the way they are combined.' Only a more specific principle, that of strong compositionality, would seem to be violated: 'The meaning of a complex expression is only a function of the meaning of its *immediate parts* and the way they are combined.' This stronger principle is sometimes adopted as a methodological constraint (cf. Montague 1970, Lasnik 1995).<sup>30</sup> But as stressed by Hodges (2005), there is no deep reason to insist that the semantics of natural language should respect it, and there is in fact evidence that it does not do so.

*Nota bene:* while consonant with them, the truth-conditions just proposed do not require the adoption of Frege's views about counting.<sup>31</sup> They only demand that the truth-conditions of the sentence depend on the internal structure of the subject.

## 2.9. The expression *is one of*

Higginbotham and Schein (1989), Schein (1993), Higginbotham (1998), and Yi (1999, 2005) argue that an approach to plurals in terms of sums fails to do justice to the semantic properties of the expression *is one of*.

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<sup>29</sup> Timothy Smiley raised this objection. See next section for an answer to it that denies that we can meaningfully use the notions of Link's "individual mereology" in the object language.

<sup>30</sup> It is also referred to as the principle of *direct compositionality* and as the *rule-to-rule hypothesis*.

<sup>31</sup> Blanchette (1999) argues that Frege's views on counting and Geach's on relative identity are incompatible. In her discussion, she suggests that Frege could attribute the following meaning to the sentence  $a_1, a_2, \dots, \text{and } a_n$  are  $m$ :

There are  $m$  distinct things falling under the concept IDENTICAL WITH  $A_1$  OR  $A_2$  ... OR  $A_n$ .

Let us start with Higginbotham and Schein's argument, focusing on the presentation in Schein (1993). A semantics of plurals should validate the following equivalence, for any count noun  $N$ :

$x$  is one of the  $Ns$  is true if and only if  $x$  is an  $N$  is true.

This raises the question of what truth-conditions mereological singularism should ascribe to the sentence on the left.

A first suggestion might be as follows:

$x$  is one of the  $Ns$  is true if and only if  $x$  is part of the sum of the  $Ns$  (formally, if and only if  $x \leq \sigma[x / Nx]$ ).

Suppose that we used the classical notion of parthood. If John is one of the boys, then any part of John, like John's arm, is also a part of the sum of the boys (by transitivity of the parthood relation), and we should infer that John's arm is also one of the boys.

So if we want better results, a restriction must be put on which parts are relevant. According to Schein, the most promising solution for the mereological singularist is to use Link's mereological notions. Simply using the notion of individual part would not be enough, for a similar reason as above. But in Link (1983)'s system, each count noun denotes a set of individuals that are atomic for the relation of individual part (i.e. individuals that have no proper individual part). Hence, the following may be proposed:

$x$  is one of the  $Ns$  is true if and only if  $x$  is an atomic individual part of the individual sum of the  $Ns$ .

(Formally:  $x$  is one of the  $Ns$  is true if and only if  $x \leq_i \sigma_i[x / Nx] \wedge \text{atomic}_i(x)$ . On the right-hand side, the index  $i$  indicates that the notions employed are those of Link's individual mereology.)

Given Link's system, this will validate the above bi-conditional. But there is a hitch. The semantics would now seem to have become inconsistent. To see this, let us apply the truth-conditions just given to the count noun *individual sum* itself:

$x$  is one of the *individual sums* is true if and only if  $x$  is an atomic individual part of the individual sum of the individual sums.

This says that an individual sum that is not atomic is not one of the individual sums! But this is false as soon as the domain of discourse contains more than two distinct atoms. The closure condition in Link's system indeed ensures that their individual sum will also be part of the domain, and this sum will not be atomic.

The mereological singularist may answer this argument as follows. The contradiction arises because two premises are accepted:

- Link's system should be used, and in particular, for any count expression  $N$ , the denotation of  $N$  is a set of individual atoms;
- the technical expression *individual sum* is a count noun that can be used in the object language.

From this, it follows directly that the term *individual sum* should denote a set of individual atoms, contrary to the rest of Link's system, which ensures that many individual sums are not atomic.

One possibility is therefore to deny that the expressions corresponding to the semantic notions used to analyze plurals in English are part of the object language. After all, ordinary English does not contain terms like *mereological sum*. Only the metalanguage used by the theorist does. However, how is this metalanguage itself understood? When moving from the object language to the metalanguage, aren't we just adding a bunch of new count nouns (*individual part*, *individual sum*, etc.) to the object language, together

with axioms governing their behavior? To this, the mereological singularist can answer that what precedes precisely shows that this addition is not innocent, appearances notwithstanding. The proposed semantics of plurals proceeds by associating to any two or more objects of the domain of quantification a different object, their individual sum. Therefore, by its very nature, the specialized vocabulary used in the metalanguage cannot apply to itself. When doing the semantics of the metalanguage, a different, meta-metalanguage must be used, with a new battery of mereological predicates. And so ad infinitum. This would simply be a fact about the semantics of plurals. Why should such a hierarchy be more problematic than the hierarchies postulated, among others, by the friends of plural logic (Williamson 2003, Rayo 2006)?

Another possibility is to deny that the denotation of a count expression must be a set of individual atoms. Furthermore, it may be argued that *x is one of the Ns* is in fact shorthand for *x is one N of the Ns*, and that:

*x is one N of the Ns* is true if and only if *x* is an *N* and *x* is part of the sum of the *Ns*. Since no single relation translates *is one of* in all contexts, no contradiction arises.

*Nota bene:* though incompatible with Link's exact system, this suggestion is consistent with a variant of it, where the requirement that the denotation of a count expression be a set of individual atoms is simply dropped. Indeed, it is enough to require that the following closure condition on the denotation of any count noun *N* be satisfied: if there is an *N*, then there is the sum of the *Ns*. This ensures that, given the things that *N* is true of, the domain of quantification contains all the sums of things that are *N* (these various sums then form a sup lattice).

At this point, it is interesting to turn to Yi (1999, 2005)'s objection, since he focuses on a somewhat different case. Take Whitehead and Russell. They have a mereological sum. Let us stipulate that the proper name *Genie* refers to this sum. Then consider the sentence *Genie is one of Frege and Genie*. Given what *is one of* means, we know that the sentence is true just in case *Genie* is identical with *Frege* or *Genie*. And so the sentence is true. Take now the sentence *Genie is one of Frege and Whitehead and Russell*. Given what *is one of* means, we know that the sentence is true just in case *Genie* is identical with *Frege*, *Whitehead* or *Russell*. And so the sentence is false. However, the sum of *Frege* and *Genie* is identical with the sum of *Frege* and *Whitehead and Russell*. So, in a semantics of plurals based on sums, the two sentences should receive the same truth-value.

A first reply would be to say that Yi's examples are ungrammatical. Indeed, several native speakers found sentences like the above, and several variations on them, extremely odd.<sup>32</sup> Moreover, an inquiry on Google returns no instance of sentences of this kind (except from Yi's own paper or citations of it). For whatever reason, it seems that noun phrases conjoined by *and* cannot be arguments of *is one of*. However, Yi's argument can easily be rephrased in a grammatical way, since *is one of them* is well formed (along with *is one of these*, *is one of those* and *is one of the Ms*, where *M* is a count noun):

*Consider Frege and Genie. Genie is one of them, of course.*

*Consider now Frege, Whitehead and Russell. Genie is one of them.*

Both stretches of discourse are well formed. In the second one, the sentence *Genie is one of them* is intuitively false. Since the oddness of *is one of Frege and Genie* does not invalidate the argument, we will ignore it in what follows.

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<sup>32</sup> Thanks to Dick Carter, Brendan Gillon, and Jerry Levinson for their judgments.



One might also worry, as a linguist, that our intuitions concerning such sentences are skewed by the fact that the meaning of the proper name *Genie* is stipulated and artificial. However, the attested proper name *Benelux* would seem to have a similar meaning. It may be argued that it denotes the sum of Belgium, Netherlands, and Luxemburg. And it would certainly be false to say that Benelux is one of France, Belgium, Netherlands, and Luxemburg!<sup>33</sup>

What if one used Link's notion of individual sum? The sentence  $PN_0$  is one of  $NP_1 \dots$  and  $NP_k$  would be true just in case what  $PN_0$  denotes is an individual part of the individual sum denoted by  $NP_1 \dots$  and  $NP_k$ . According to this new version of the theory:

*Genie is one of Frege and Genie* is true if and only if  $g \leq_i (f +_i g)$

*Genie is one of Frege and Whitehead and Russell* is true if and only if  $g \leq_i (f +_i w +_i r)$

If the example is set up so that *Genie* ( $g$ ) is the individual sum of *Whitehead* ( $w$ ) and *Russell* ( $r$ ), then both sentences are predicted to have the same truth-value, contrary to the intuitions of naïve speakers.<sup>34</sup> Here, the mereological singularist could argue, similarly as above, that by considering the proper name *Genie*, we have moved into the metalanguage of the theorist, and that a new battery of mereological predicates is needed to interpret that language.

Another possibility is to claim that the data presented by Yi concerning *is one of* show that this expression is sensitive to the linguistic properties of the expressions it is combined with. First, the expression requires its first argument to be semantically singular (*\*Whitehead and Russell are one of these men*), and it requires its second argument to be plural (*\*Whitehead is one of this man*). Second, the interpretation of *is one of* is sensitive to the linguistic structure of its arguments. Notably, a sentence of the form  $PN_0$  is one of  $PN_1$  and  $PN_2$  is true just in case  $PN_0$  is identical with  $PN_1$  or  $PN_2$  (where  $PN_0$ ,  $PN_1$  and  $PN_2$  are three singular proper names). Likewise,  $PN_0$  is one of  $PN_1$  and  $PN_2$  and  $PN_3$  is true just in case  $PN_0$  is identical with  $PN_1$ ,  $PN_2$  or  $PN_3$ .

Now, *Genie* is, by stipulation, a singular proper name, while *Whitehead and Russell* is a plural noun phrase. Therefore, there is no reason to expect that we should be able to replace the expression *Whitehead and Russell* by *Genie* when it is an argument of the expression *is one of*. On the contrary, given what has just been said, it is clear that  $PN_0$  is one of  $PN_1$  and *Genie* and  $PN_0$  is one of  $PN_1$  and *Whitehead and Russell* will always have different truth conditions. These truth conditions are easily stated, as was done above.

It should be noted that this position does not attribute a unitary meaning to the expression *is one of*. It only assigns truth-conditions to a sentence-schema:

$PN_0$  is one of  $PN_1 \dots$  and  $PN_k$  is true just in case  $PN_0$  is identical with  $PN_1 \dots$  or  $PN_k$ .

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<sup>33</sup> This was remarked by Brendan Gillon.

<sup>34</sup> One might propose that the meaning of singular, grammatical number requires that a singular term only denote an individual atom. This example would then be ruled out. But such a requirement seems to be unmotivated. Suppose that the universe of discourse contains just two individual atoms,  $a$  and  $b$ , and their individual sum,  $c$ . In Link's theory, an atom is a sum, so  $a$ ,  $b$  and  $c$  are individual sums. Therefore,  $c$  is one of the individual sums that are present in the universe of discourse. All of what has just been said seems to be true, and to be expressed in plain English. In other words, given Link's theory, it seems a priori possible to use a singular term to refer to any individual sum.

The situation appears to be very similar to that encountered in the previous section. In general, we might consider discourses of the following forms (where  $Q$  is any verb):

$NP_1 \dots$  and  $NP_k$   $Q$ ed. *They were n.*

$NP_1 \dots$  and  $NP_k$   $Q$ ed.  $PN_0$  was one of them.

In each case, we need a method for deciding what is to count as one. What our discussion shows is that there is at best a non-unitary method for doing so, one that takes the internal structure of  $NP_1 \dots$  and  $NP_k$  into account (*they* and *them* being anaphoric on this complex linguistic expression). Is that a problem, or just a fact about the semantics of these expressions?

### 3. Conclusion

We have discussed various arguments against the idea that mereological sums can serve as the semantic values of plurals. As an aid to memory, let us give the following summary of the ground we covered.

#### 1) Do abstract objects have sums?

Mereology is characterized by certain axioms. Such axioms are a priori subject-neutral. It is not necessary to limit their application to the realm of concrete or mono-categorical entities.

#### 2) Plurals do not carry ontological commitment to mereological sums

A medium of representation typically carries its own commitments, distinct from those of what is represented. The commitments of each just need to be distinguished.

#### 3) John and Mary met, but their mereological sum did not

The best option seems to be to bite the bullet: the sentence *The sum of John and Mary met* is odd, but only because we are using our intuitions about the normal uses of the expression *sum* and *met*, while employing a partly new language.

#### 4) Two men wrote a book, but their molecules did not

Using a single mereology forces one to identify the men and their parts. One solution is to employ (at least) two distinct mereologies, as suggested by Link. Another may be to bite the bullet, just as for 3).

#### 5) The piles of grains are scattered, though the grains need not be

Sentences containing plurals are liable to distributive, collective and intermediate construals. Once this is taken into account, the purported problem disappears.

#### 6) The non-atoms are not the atoms

Two solutions: i) maintain that the sentence makes sense only under a distributive construal of each plural noun phrase (as was argued in the main body of the text); ii) argue on the contrary that the sentence may also receive a collective construal, under which it is in fact true that *the non-atoms are the atoms* (cf. note 16).

#### 7) Regions a and b overlap, but a minus b and b do not overlap

The best option seems to claim that *overlap* is a covert reciprocal in *a and b overlap*, and that the rules of English indicate that this should be interpreted as *a overlaps b, and b overlaps a*.

#### 8) Counting

The singularist has two main options. One is to consider a hierarchy of metalanguages, each with its own mereological predicates. Another is to claim that the truth-conditions of a sentence of the form  $NP_1 \dots$  and  $NP_k$  are  $n$  are dependent on the internal structure of the subject.

#### 9) The expression *is one of*

This case is analogous to the previous one, and it leads to similar moves.

Our main question was: how strong are the arguments against mereological singularism? The first seven seem to receive relatively easy answers. In particular, in the case of 2), 3) and 4), the oddness of the metalanguage (concerning linguistic intuitions or ontological commitment) simply does not disqualify the proposed analysis.

In the case of 8) and 9), the difficulties are more significant. The adoption of two mereologies, as suggested by Link, looks at first promising, but it does not work if one uses the expression *individual sum* in the object language. So one possible solution for the mereological singularist consists in denying that this is possible, and in considering an infinite hierarchy of metalanguages. Another consists in claiming that the semantics of *to be n* or *to be one of* is sensitive to the internal structure of the relevant argument. Both options may be seen as having a certain cost. The latter requires abandoning the methodological hypothesis that English is strongly compositional,<sup>35</sup> while the former has no basis in our intuitions: we have the impression of using the plural in the same way when we are talking of cats and when we are talking of individual sums.

These difficulties arise in particular for Link's theory, something that is a bit surprising in light of the following facts. Whenever there are some individuals, Link associates to them a certain object, their individual sum. As Link (1998) makes clear, the structure needed for this is that of a complete, atomic Boolean algebra minus its zero. Now, plural logic can be developed with a single type of variables, namely plural ones. The axioms of such a logic are then those of a complete atomic Boolean algebra minus its zero. So, from an axiomatic point of view, there is a strict equivalence between an atomic mereology and a plural logic with only plural variables.<sup>36</sup> Why, then, would they differ when employed to characterize the truth-conditions of plural sentences?

The only and essential difference between these theories is one of ideology. Link's semantics says that every plural expression denotes an object, a certain individual sum. Therefore, it may seem that it should be possible to talk about these objects themselves, as *individual sums*, and that the semantics adopted for the plural should apply to this plural count noun. But as we saw in sections §2.8 and §2.9, Link's semantics does not work well when it is applied to the expression *individual sums*. In plural logic, a plural expression simply denotes several objects at once, so this difficulty does not arise.<sup>37</sup>

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<sup>35</sup> But it does not require abandoning the hypothesis that English is compositional *simpliciter*.

<sup>36</sup> See Simons (1982b), Link (1998, chapter 14) and McKay (2006, chapter 6).

<sup>37</sup> However, a difficulty does appear when we want to provide an account of logical truth and logical consequence for a language incorporating plural quantification and predication. To do so, we need to quantify over interpretations, and so we need to specify, in the metalanguage, what an interpretation is. Now, plural logic derives its impetus in great part from the assumption that it is possible to quantify over absolutely everything there is. This assumption ensures, for instance, that a systematic semantics of plurals based upon sets will run into Russell's paradox. But it also has the consequence that the metalanguage used to specify interpretations must be strictly stronger than the object language (Williamson 2003, Rayo 2006). As a result, the friend of plural logic is led to consider a hierarchy of languages, in which each language is suitable as a metalanguage for its immediate predecessor, but requires a strictly stronger metalanguage for an account of logical truth and logical consequence.

#### 4. References

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