Mass nouns and plural logic [6 pages abstract]

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MASS NOUNS AND PLURAL LOGIC

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Abstract There are two main approaches to the semantics of mass nouns (cf. Bunt 1985, Pelletier & Schubert 1989). One uses sets as their semantic values, the other uses mereological sums. Both face difficult problems, notably with sentences like *The gold on the table weighs seven ounces* (Bunt 1985) and *The clay that made up those three bowls is identical with the clay that now makes up these two statues* (cf. Cartwright 1965). We propose a new theory able to solve these problems, in a framework different from predicate logic, called plural logic. Our semantics is faithful to the intuition that, if there are eight pieces of silverware on a table, the speaker refers to eight things at once when he says: *The silverware that is on the table comes from Italy*.

1. Introduction

Former approaches to the semantics of mass nouns either use sets or mereological sums as their semantic values. They face serious difficulties (cf. Bunt 1985, Pelletier & Schubert 1989). To address those, we propose a new theory, in a framework different from predicate logic, called plural logic.

2. Former approaches

2.1. The set approach to mass nouns

The set approach to mass nouns (e.g. Strawson 1959, Laycock 1972) treats them as predicates. (1) *This is wine* is true if and only if I(*this*) $\in$ I(*wine*), where I is the interpretation function, I(*this*) is what is demonstrated, and I(*wine*) is the set of everything that can be said to be wine.

The question is then how to treat mass nouns in definite descriptions, as in (2) *The gold on the table weighs fifty grams* (Bunt 1985). If the description denotes the set having for element anything that is gold on the table, then how can we evaluate the truth of the sentence? It would not do to give the sum of all weights. So we must impose restrictions on the elements of the set I(*the gold on the table*).

Now comes a second difficulty concerning identity over time. Consider: (3) *The clay that made up those three bowls is identical with the clay that now makes up these two statues*. Which set could make I(*the clay that made up those three bowls*) = I(*the clay that now makes up these two statues*) true?

What about the set of all minimal parts of gold, i.e. the set of all the instances of gold that have no other instance of gold as part? However, with mass nouns like *garbage*, it is not clear what the minimal parts would be (cf. Pelletier & Schubert). Moreover, mass nouns like *time* and *space* do not seem to have minimal parts. So the semantics of mass nouns should not require them to have some.

2.2. The mereological approach to mass nouns

The mereological (or lattice-theoretic) approach to mass nouns (e.g. Link 1983, Gillon 1992) focuses first on mass nouns in definite descriptions. When *M* is a mass noun, it takes the define noun phrase *the M that Qs* to refer to the mereological sum of everything that is some M that Qs. It is such a sum that is weighted in sentence (2), and whose identity over time is asserted in (3).

But (1) must still be dealt with. The proposal is this. (1) is true if and only if I(*this*) ≤ I(*wine*), where ≤ is the relation of parthood, I(*this*) is what is demonstrated, and I(*wine*) is the mereological sum of everything that is wine.

This yields a new problem with minimal parts. An atom of hydrogen is not water, though it is part of a molecule of H2O . More strikingly, a leg of a chair is not furniture, though it is part of a chair, and a chair is some furniture (Bunt 1985).

Moreover, sentences containing mass nouns (just like sentences containing plurals) are liable to so-called collective, distributive, and intermediate construals (cf. Gillon). Thus, (4) *This silverware costs a hundred euros* may be true if the silverware costs, all together, a hundred euros (collective construal). It may be true if each piece of silverware costs a hundred euros (distributive construal). It may also be true if the silverware demonstrated consists in two sets of silverware, each set costing a hundred euros (intermediate construal). To capture these construals, a notion of covering akin to that proposed by Gillon is needed, and to express this notion, the apparatus of sets, or something as expressive, is required: mereological sums are not enough. Gillon's approach is thus mixed, being based on mereological sums, but using sets for coverings. (NB: Gillon uses the term *aggregation* instead of *covering*.)

The mereological approach (be it Link's or Gillon's) faces two additional, independent problems. Consider a sentence that could be taken from *Animal farm*, George Orwell's novel: (5) *The livestock met on the hill.* The approach takes it to be true if and only if the mereological sum of the livestock met on the hill. But the right-hand side of this bi-conditional is in fact very odd: the English predicate *meet* does not seem to apply to mereological sums.

Finally, in a given circumstance, it may be that the mereological sum of the M is identical with the sum of the N (where *M* and *N* are two mass nouns), yet the sentences *The M Qs* and *The N Qs* have, intuitively, opposite truth-values. Thus, suppose that some wood of a given kind (e.g. some elm) is used to make furniture of different styles. Then intuitively, the sentence (6) *The furniture is heterogeneous* would be true, while (7) *The wood is heterogeneous* would be false. But furniture and wood have the same sum, so the theory predicts that (6) and (7) should have the same truth-value.

3. A new approach based on plural logic

We propose a new approach to solve these difficulties. Its starting point is the intuition that, if there are three solid bits of gold on the table, then the subject noun phrase of sentence (2) refers to three things at once. It is these three things that are jointly weighted. This makes mass nouns very similar to plurals, though not identical with them. (NB: Mass nouns are invariable in grammatical number. Therefore, it is coherent to suppose that number has no semantic value with these nouns.[1])

Now, Oliver & Smiley (2001) and Rayo (2002) have shown that, if we acknowledge that it is possible to quantify over absolutely everything there is, the semantics of plurals should not be characterized using predicate logic and sets. Indeed, some intelligible sentences containing plurals would be represented in a contradictory way. On the other hand, if one employed mereological sums and predicate logic, the semantics of plurals would turn out to be too weak. To avoid this, they propose to use plural logic. Plural logic contains plural terms (like 'as') that can refer to several things at once (to a "plurality of things"). A sentence where the predicate applies to a plural term is true when the objects that interpret the term satisfy together (collectively) the predicate. Plural logic also contains "superplural" terms (like 'css'), which can

[1] A difference between English and French confirms this. In English, mass nouns tolerate only determiners that can also be used with plurals: *some, all, any, the*. Not so in French, where the determiner must be singular: *de l'or / *des or* (some gold), *tout or / *tous or* (all / any gold), *l'or / *les or* (the gold).

Moreover, mass expressions and plurals often seem to be co-referential: *The silverware is in the drawer / The pieces of silverware are in the drawer.* Strikingly, French possesses both a mass noun (*mobilier*) and a count noun (*meuble*) that refer to pieces of furniture.

Like their invariability with respect to number, all these data suggest that grammatical number has no semantic implications for mass nouns.

refer to "several pluralities" at once.[2]

We use plural logic to have a common framework for mass nouns and plurals, in which a mass expression may refer to several things at once. The denotation of a mass noun *M* is identified by a plural term, 'ds': some thing is M just in case it is one of the ds. Then sentence (1) is true just in case the things referred to by *this* are among the ds.

To deal with examples like (6) and (7), we first remark that if the furniture is cut into pieces, it is destroyed, but the wood remains. So a semantics of mass nouns should not identify furniture and wood. Typically, the parthood relation is taken to be extensional, which forces the identification of the two (Parsons 1970). To avoid this, we make the parthood relation a partial ordering, but require it to satisfy the axiom of strong complementation (cf. Simons 1987) only relative to any given mass noun *M*:
$\forall u\, \forall v\, ((Mu \wedge Mv \wedge \neg\, (u \leq v)) \rightarrow \exists\, x\, (Mx \wedge x \leq u \wedge \neg\exists y\, (y \leq x \wedge y \leq v)))$
In this way, our theory can coherently deny that the furniture is identical to the wood that makes it up.

The remaining problems concern definite descriptions. They are dealt with by requiring mass nouns to satisfy additional axioms (which are stated precisely in the *Appendix*). A definite mass noun phrase like *the gold on the table* refers to several things, the as. The axioms say that each of them is M (e.g., each is gold), self-connected (of a single piece, cf. Casati & Varzi 1999), and maximal for the relation of parthood. Each is, for instance, one of the solid bits of gold that are on the table.

The interpretation of (2) is then relative to the choice of a covering of the as. A covering of the as is given by a "plurality of pluralities", the css. The sentence says, of each "plurality" of this covering, that the things that make up this "plurality" weigh seven ounces together. Among the various construals of the sentence, the collective one is of course the most salient (the other construals requiring much more context to become available). It is obtained when the covering contains only one "plurality", the as themselves. The sentence says that the as (the solid bits of gold) weigh seven ounces together.

This applies straightforwardly to (4), which indeed motivated the need for coverings, and to (5). It also yields a satisfactory semantics for (3). The sentence is made true by a suitable choice of covering, the css, each of which is some clay that has retained its identity over time. (This does not require the existence of minimal parts.) At a previous time, the css together made up three bowls. They have been rearranged,

[2] This is only loosely speaking. In plural logic, no object is a "plurality" or a "plurality of pluralities". But plural logic contains stronger forms of reference than singular reference: plural reference (to several things at once), and superplural reference (on the latter see Rayo 2006 and Linnebo & Nicolas 2008).

shuffled, so as to now make up two statues.

Moreover, the framework also applies to quantified statements, like (8) *All phosphorus is either red or black*. Roeper (1983) thought that this kind of example was problematic for set-theoretic approaches of mass nouns. In the present account, the sentence is made true by a covering that is a bi-partition of all the phosphorus, such that the bits of phosphorus in one "plurality" of this covering are red, while the other bits of phosphorus are black.

4. Conclusion

Set-theoretic and mereological approaches to the semantics of mass nouns face difficult problems, notably with sentences (1) to (8). Using plural logic, we have proposed an account able to solve these difficulties. The key was to define a notion of *covering* applicable to things that do not have a mereological sum. This allowed us to devise a semantics where mass nouns may refer to several things at once, while dealing satisfactorily with these difficulties. (For considerably more details from a partly different perspective, and for acknowledgements, see Nicolas, manuscript.) This semantics is faithful to the intuition that, if there are eight pieces of silverware on a table, the speaker refers to eight things at once when he says (9) *The silverware that is on the table comes from Italy*, and that in this particular case, he might as well have said (10) *The pieces of silverware that are on the table come from Italy*.

5. Appendix: main technical notions employed in our semantics for mass nouns

Maximal elements:

$ys = \max[zs / Qzs] \equiv_{def} \forall zs\, \forall u\, ((Qzs \wedge u \angle zs) \rightarrow \exists v\, (v \angle ys \wedge u \leq v))$

$\wedge\, \forall v\, (v \angle ys \rightarrow \exists zs\, (v \angle zs \wedge Qzs)) \wedge \neg(\exists u\, \exists v\, (u \angle ys \wedge v \angle ys \wedge u \neq v \wedge Ouv))$

Among the zs that Q, the ys are the maximal elements for the relation of parthood.

Axioms:

$\forall zs\, (Mzs \rightarrow \forall x\, ((x \angle zs) \rightarrow Mx))$ If some things are M, each of them is M.

$\forall x\, (Mx \rightarrow \text{self-connected}(x))$ If something is M, it is self-connected.

$\exists zs\, (Mzs \wedge Qzs) \rightarrow \exists ys\, (ys = \max[zs / Mzs \wedge Qzs])$

Guarantees that *the gold on the table* refers to the three solid bits of gold on the table.

Covering: the css are a covering of the as just in case:

i) Any of the css is M: $\forall x\, (x \angle^{\circ} css \rightarrow Mx)$

ii) For anything x, x overlaps one of the css iff x overlaps one of the as:

$\forall x\, (\exists v\, (v \angle^{\circ} css \wedge Oxv) \leftrightarrow \exists w\, (w \angle as \wedge Oxw))$

Truth-conditions of a sentence of the form 'The M that Qs Ps':
Let the as be the denotation of *the M that Qs*. They satisfy: as = max[zs / Mzs $\wedge$ Qzs]. The interpretation of the sentence depends on the choice of a covering of the as. Let the css be the chosen covering. The sentence is true if and only if the predicate P applies to "each plurality" in the covering: $\forall$ys (ys $\angle$' css $\rightarrow$ Pys)

Truth-conditions of 'All M Ps':
Let the ds be the denotation of the mass noun *M*. The interpretation of the sentence is relative to the choice of a covering of the ds containing at least two "pluralities". Let the css be the chosen covering. The sentence is true if and only if the predicate P applies to "each plurality" in the covering: $\forall$ys (ys $\angle$' css $\rightarrow$ Pys)

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