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June 2010

To appear in *Thinking and Reasoning*

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Abstract

A study is reported testing two hypotheses about a close parallel relation between indicative conditionals, *if A then B*, and conditional bets, *I bet you that if A then B*. The first is that both the indicative conditional and the conditional bet are related to the conditional probability, P(B|A). The second is that de Finetti's three-valued truth table has psychological reality for both types of conditional – *true*, *false*, or *void* for indicative conditionals and *win*, *lose* or *void* for conditional bets. The participants were presented with an array of chips in two different colours and two different shapes, and an indicative conditional or a conditional bet about a random chip. They had to make judgments in two conditions: either about the chances of making the indicative conditional true or false or about the chances of winning or losing the conditional bet. The observed distributions of responses in the two conditions were generally related to the conditional probability, supporting the first hypothesis. In addition, a majority of participants in further conditions chose the third option, “void”, when the antecedent of the conditional was false, supporting the second hypothesis.

Betting on conditionals

There is a new probabilistic, or Bayesian, paradigm in the psychology of reasoning, with new *probability theories* of the natural language indicative conditional (Evans, 2007; Evans & Over, 2004; Chater & Oaksford, 2009; Oaksford & Chater, 2001, 2007, 2009; Over, 2009; Manktelow, Over, & Elqayam, in press; Pfeifer & Kleiter, 2006, 2010). The new theories imply two hypotheses about a close parallel relationship between indicative conditionals in natural language and conditional bets. The first hypothesis is that indicative conditionals and conditional bets are related to the conditional probability. The second hypothesis is that a classification of *true*, *false*, and *void* for indicative conditionals is paralleled by one of *win*, *lose*, and *void* for conditional bets. These hypotheses were first implied by the founders of contemporary subjective probability theory (de Finetti, 1937/1964; Ramsey, 1931/1990). The central goal of this paper is to explain the theoretical reasons why the parallel relationship is expected and to study in an experiment whether it exists in people's judgments.

People use many different types of conditionals in natural language (Evans & Over, 2004). Psychologists have followed philosophers in studying, for example, causal conditionals and counterfactuals (Over, Hadjichristidis, Evans, Handley, & Sloman, 2007; Thompson & Byrne, 2002). The new category of utility conditionals has also been identified (Bonnefon, 2009). The widest and most basic category is that of an indicative conditional, such as:

(1) If unemployment increases in the coming year, then crime will too.

There could be an argument about how much confidence to have in (1). We could justify (1) by referring to published statistics on unemployment and crime, but the relevance of this data for the coming year could be disputed. As the argument about the confidence to have in (1) went back and forth, we might slip easily into making a conditional bet:

(2) I bet that, if unemployment increases in the coming year, crime will too.

Slipping back and forth between conditionals like (1) and (2) in a discussion is not only a common linguistic practice. There are also theoretical reasons in probability theories of the indicative conditional, *if A then B*, for holding that there is a close relation between this conditional and a conditional bet. According to these theories, there is the following parallel relationship between (1) and (2). The conditional bet (2) is won in the case that unemployment increases and crime does too, *A & B* holds, and lost when unemployment increases and crime does not, *A & not-B* holds. When unemployment does not increase, in the *not-A & B* and *not-A & not-B* cases, the bet is called off and no one wins or loses. Such a “called off” conditional bet is termed *void*, but it remains a good bet to have made if the conditional probability of an increase in crime given an unemployment increase, P(B|A), was high. The indicative conditional (1) is shown to be true when unemployment increases and crime does too, *A & B*, and shown to be false when unemployment increases and crime does not, *A & not-B*. We would naturally say that we will “win” the argument about (1) if *A & B* holds and “lose” it if *A & not-B* holds. In the *not-A & B* and *not-A & not-B* cases, (1) is not shown to be true and not shown to be false. It could still be said that (1) was a justified assertion just in case P(B|A) was high, but we would tend to lose interest in the indicative (1) as we became more and more confident of *not-A.* As this happened, we would become more inclined to use a counterfactual, such as “If A had been then B would have been”. When we do prefer to assert the counterfactual because *not-A* is known, the indicative (1) is neither true nor false in probability theories and can also be termed *void*.

Recent experimental research strongly confirms that people judge P(if A then B) = P(B|A) for indicative conditionals in natural language. A conditional *if A then B* for which P(if A then B) = P(B|A) is sometimes called a *probability conditional* (Adams, 1998). There is much evidence against the alternative claim that the probability of an indicative conditional is the probability of the *material conditional* of extensional logic, P(not-A or B), as implied by *mental model theory* (Johnson-Laird & Byrne, 2002; Byrne & Johnson-Laird, 2009, 2010). The modal response is to give the conditional probability as the probability of the conditional in experiments on a very wide range of indicative conditionals. These include “basic” indicative conditionals about simple frequency distributions and indicative conditionals like (1) that could be called “causal” (Evans, Handley, & Over, 2003; Fugard, Pfeifer, Mayerhofer, & Kleiter, 2010; Oberauer & Wilhelm, 2003; Over & Evans, 2003; Over et al., 2007). Many results in the psychology of conditional reasoning can also be explained by holding that indicative conditionals are probability conditionals (Evans & Over, 2004; Oaksford & Chater, 2007, 2009; Over, Evans, & Elqayam, 2010; Pfeifer & Kleiter, 2006, 2010).

In contrast, very little psychological research has been done on conditional bets and their relation to probability judgments about indicative conditionals. This is surprising given the similarities between these conditionals implied by probability theories of the conditional. There are differences between these psychological theories of the conditional (compare, for example, Evans & Over, 2004, Oaksford &

Chater, 2007, and Pfeifer & Kleiter, 2010), but all the theories entail that a bet on an indicative conditional is a bet on a probability conditional. The similarities have been mainly described in normative theories. Ramsey (1931/1990) and de Finetti (1931, 1937/1964) were the first to argue for a close relation between indicative conditionals, conditional bets, and conditional probability.

Ramsey (1931/1990) proposed a method for evaluating indicative conditionals that has long had a major impact in philosophical logic (Bennett, 2003) and more recently in psychology (Evans & Over, 2004; Oaksford & Chater, 2007). It is known as the *Ramsey test* for subjective probability judgments about conditionals: how confidently to believe or assert an indicative conditional. He considered two people arguing about a conditional like (1). He suggested that they proceed by adding A "hypothetically to their stock of knowledge and arguing on that basis about" B, and that "they are fixing their degrees of belief" in B given A, P(B|A). Should A turn out to be false, so *not-A* is the case, he held that the degree of belief in the indicative conditional is "void". The Ramsey test thus introduces a probability conditional, and it can clearly be taken as a psychological hypothesis about how people evaluate natural language conditionals (Evans & Over, 2004). It can be compared to the simulation heuristic, for evaluating an epistemic relation between A and B, by "simulating" A and then seeing to what extent B follows (Evans, 2007; Kahneman & Tversky, 1979).

The work of de Finetti (1936, 1937/1964) on conditionals has so far had less direct impact than Ramsey's on the recent psychology of conditionals, but equally supports the relation between the indicative conditional and conditional probability. In his analysis of a probability conditional, which he called the *conditional event*, he went beyond the two valued semantics for non-conditional propositions, or "events", like A and B, which only have the two values of truth (T) and falsity (F). For the conditional, there is also the void case, which in philosophical logic is described as a *truth value gap* (Bennett, 2003; Evans & Over, 2004). He defined the conditional event of B conditioned on A, terming it also a “tri-event” denoted by B|A. The tri-event B|A can be described using the four rows of the truth table for A and B. Using N (Null) for the void case, we have:

TT, when A is true and B is true, B|A is T

TF, when A is true and B is false, B|A is F

FT, when A is false and B is true, B|A is N

FF, when A is false and B is false, B|A is N

The N classification (later on denoted ∅ - see de Finetti 1967, 2008) does not distinguish between the two rows where A is false and so *not-A* holds. The above could be called the *de Finetti table* and compared to the so-called “defective” truth table that matches people’s evaluations of conditionals (Evans & Over, 2004; Manktelow et al., in press). People judge that *not-A* cases are “irrelevant” to the truth or falsity of an indicative conditional (Evans & Over, 2004). Notice that, by the de Finetti table, such a judgment is not the result of ignorance, as if the conditional had some unknown or indeterminate truth value. It is rather the result of knowing that *not-A* holds.

Following de Finetti, we can also think of the de Finetti table as a description of the outcomes of a conditional bet, in which T is winning, F is losing, and N is the void case. We can use our example of an argument about (1), leading to the conditional bet (2), to illustrate the meaning of the table. We win an argument to support our assertion of (1), and the conditional bet (2) we made as part of it, when unemployment and crime both increase in the coming year, in the TT row, and we lose it when unemployment increases but crime does not, in the TF row. When unemployment does not increase, in the FT or the FF row, we do not win or lose the argument or the bet. Again, saying the bet is “void” does not mean that it has some unknown value of being won, lost, or something in between; it would be the result of knowing that *not-A* holds.

Given the de Finetti table, we should notice a possible ambiguity in questions about (1) and (2). Suppose we are asked for our confidence in the truth of (1). The word *true* here could be taken as pleonastic (Edgington, 2003; Over et al., 2007), making the question simply about our confidence in (1), with the answer P(B|A) by the Ramsey test. The question can be also taken to be about the possibility that (1) makes an assertion that is true or false. The *not-A* cases are irrelevant to the question because the conditional is neither true nor false in these cases. This presupposition makes only rows TT and TF relevant to the question, and the answer to it is again P(B|A). Similarly, we might be asked for our confidence that we will win the bet (2). This question, with its reference to a bet, can be taken to be about the possibility that (2) does make a bet - that the void cases, when there is no proper “bet”, are not to be considered. That makes the *not-A* outcomes irrelevant to the question. This presupposition again makes only rows TT and TF relevant to the question, and the answer to it is P(B|A). In contrast, if all the rows, TT, TF, FT, and FF, are judged relevant to the questions about (1) and (2), the answer would be the conjunctive probability, P(A & B). We argue that, if probability theories of the indicative conditional, appealing to the Ramsey test and de Finetti table, are correct, the presupposition that people will tend to make is that only rows TT and TF are relevant to the questions.

Consider again what psychologists have long called the “defective” truth table, which we are calling the de Finetti table. As noted above, people tend to judge the *not-A* outcomes, *not-A & B* and *not-A & not-B*, as “irrelevant” to the truth or falsity of *if A then B* (Evans & Over, 2004). There is evidence that people also consider these cases irrelevant when they are asked about the probability of truth, or the probability of falsity, of *if A then B*. The modal response to the probability of truth question is P(B|A), and P(not-B|A) to the probability of falsity question, with the two responses summing to 1 (Evans et al., 2003). Most participants do not respond with the conjunctive probability, P(A & B), even though they judge *A & B* to be the only outcome that makes *if A then B* true (for more on the conjunctive probability response, see Evans, Handley, Neilens, & Over, 2007). In other words, they do make the supposition that a question about the probability of truth of *if A then B* concerns the probability of its truth out of the relevant outcomes, when there is no truth value gap, and a non-void assertion is made (see Fugard et al., 2010, for a study of the probability of *if A then B* that does not ask for the probability of truth). The probability of *A & B* given that there is a non-void assertion of the conditional is P((A & B)|A), which is simply P(B|A).

It is an open question, which we aim to answer in this paper, whether people will presuppose that *not-A* outcomes are similarly irrelevant to a question about the probability of winning a conditional bet on *if A then B*. The *A & B* outcome is the winning one, just as it is the truth making one. But if probability theories are correct, the *not-A* outcomes are irrelevant, because the bet is void and called off in *not-A* cases. The question about the probability of winning should be taken as opposed to the probability of losing, with the *not-A* void cases ignored as ones in which there is neither a win nor a loss. The probability of the winning outcome *A & B* given that there is a non-void conditional bet is again P((A & B)|A) = P(B|A).

Another fundamental reason to focus on P(B|A) follows from probability theories of the conditional. Consider the bet made in Figure 1. A distribution of chips is displayed, and a chip is randomly selected. Mary says, “I bet you 1 Euro that, if the chip is square, then it is black”, and Peter accepts this conditional bet at even money. It can be seen that Mary has the advantage in this bet. Its positive expected value for her can be calculated using the displayed distribution and the de Finetti table. She wins 1 Euro when *A & B* holds, loses 1 Euro when *A & not-B* holds, and neither wins nor loses, getting her 1 Euro back, when not-*A* holds and the bet is void. Therefore, her expected value for the bet, by the de Finetti table, comes from the following formula (assuming her subjective probability and utility judgements are based on the objective distribution of chips and value of 1 Euro):

$$P(A \,\&\, B)(1) + P(A \,\&\, \text{not-}B)(-1) + P(\text{not-}A)(0) = .29$$

The expected value of the bet for Mary is 29 cents. That means that, in the technical sense, it is not a *fair bet*, which has an expected value of 0 by definition. Of course, we do not believe that most of the bets we make in ordinary affairs are fair in this sense. Sensible people know that casinos have the advantage when bets are placed on roulette weeks and the like. People would usually make a bet on (1) in an argument because they thought that they had the advantage. Even so, the notion of a fair bet, the expected value of which is 0, is fundamental. If we are on one “side” of it, like Mary, we have the advantage, but if we are on the other "side", like Peter, we are at a disadvantage.

Let us now consider how much Mary would have to risk for the bet to be fair. If Peter continues to bet 1 Euro, Mary will clearly have to bet 3 Euro for the expected value to be 0 by the above formula and the frequency distribution. Supposing that the de Finetti table is the correct account of the conditional, and Mary knows the frequency distribution of the chips, betting 3 Euros to 1 should be as far as she is prepared to go. These odds of 3 to 1 correspond to 3 chances out of 4 and a probability of 0.75 that she will win the conditional bet. In a Ramsey test, this probability P(B|A) = .75 comes from focusing on the square *A* chips and then taking the ratio of square and black, *A & B*, chips to these *A* chips. Supposing that the chances of Mary's winning is the ratio of *A & B* chips to all chips, both *A* and *not-A*, giving the conjunctive probability, P(A & B) = .43, would make the conditional bet appear disadvantageous to Mary. If that were the probability of her winning, she should not be betting 50/50, equal money with Peter. But it is obviously wrong to claim that the bet as stands is disadvantageous to Mary, given that no money changes hands when a *not-A* outcome occurs. The de Finetti table correctly shows *not-A* outcomes as void.

Ramsey and de Finetti argued independently, and in a slightly different ways, that people's degree of confidence in their beliefs and assertions should be found in the bets they judge to be fair. They showed that the only way for these degrees of belief, or subjective probabilities, to be safe from a Dutch book, in which there is a necessary loss, is for them to obey the axioms of additive probability theory (see de Finetti, 1937/1964, for the formal details and the Dutch book result). And as we have explained, their work leads straight to an account of the indicative conditional as a probability conditional. Its probability is given by P(if A then B) = P(B|A), and a bet on it is a bet on such a probability conditional.

To see conditionals like (1) and (2) as essentially connected has theoretical advantages. It explains the relation between the indicative conditional and a conditional probability argued for by a number of influential theorists (Adams, 1998; Bennett, 2003). It leads immediately to an account of coherence and incoherence for conditional beliefs and assertions (Coletti & Scozzafava, 2002; Gilio, 2002). Furthermore, the analysis of conditional bets offers a justification, not only theoretical, but also intuitive for a three-valued logic for conditionals (Milne, 1997). This formalization has been used by AI researchers (Calabrese, 1994; Dubois & Prade, 1990; Goodman, Nguyen and Walker, 1991) and also by philosophers (Milne, 1996, 1997). Lastly, this approach evades a theoretical problem resulting from the view that the probability of a conditional is the conditional probability, namely Lewis's (1986) triviality results (Mura, 2009; Paneni & Scozzafava, 2004). The triviality results assume that there are no void cases: that the conditional *if A then B* is always true or false in *not-A* cases and hence never has a truth value gap. When there are truth value gaps, the probability of the conditional *if A then B* can be the conditional probability (see Evans & Over 2004, Ch. 2, on T2, with no truth value gaps, and T3, with truth value gaps, accounts of the conditional).

Ramsey and de Finetti were normative theorists. There is every reason to compare indicative conditionals and conditional bets in psychology as well, but there has been only one earlier study (in addition to Politzer & Baratgin, 2006), that has made this comparison. Oberauer and Wilhelm (2003) took the significant step of asking participants to estimate the probability that a conditional was true, and also to indicate the maximum amount that they were willing to bet on the truth of the conditional. The object was to avoid using the term “probability” in the question. The two measures were found to correlate reliably, but the coefficient was very small (about .15 on the average) and the magnitude of the probabilities differed sharply (about .50 in the former case and less than .20 in the latter). More investigation is needed.

Before we turn to our experiment, we will examine what mental model theory implies about indicative conditionals and conditional bets. This theory is the most prominent alternative to probability theories of indicative conditionals. Byrne and Johnson-Laird (2009, 2010) allege that it is “erroneous” for people to judge the probability of a conditional as the conditional probability. They connect this claim with their belief that, when phrases like *it is not the case that*, *it is necessary that*, or *it is probable that* occur at the beginning of a conditional, “…they apply only to the main clause”, so *it is necessary that if A then B* supposedly becomes *if A then it is necessary that B*. They add that the former use of *it is necessary that* applied to the whole conditional “is synonymous with” the latter use applied only to the consequent (Byrne & Johnson-Laird, 2010, p. 56). We should note immediately that *if A then it is necessary that B* has what linguists and logicians call a *scope ambiguity*. In logical terminology, what Byrne and Johnson-Laird are claiming is that the *wide scope* form *it is necessary that (if A then B)* means the same as the *narrow scope* version of *if A then it is necessary that B*, which logicians would indicate by *if A then (it is necessary that B)*.

Modal logicians would point out that Byrne and Johnson-Laird are committing a *modal fallacy*: the wide scope use of *it is necessary that* is not synonymous with the narrow scope use (Bradley & Swartz, 1979). In the wide scope statement, there has to be a necessary relation between *A* and *B*, but in the narrow scope statement, the necessity of *B* is stated to follow simply from *A*. For example, *it is necessary that (if A then A)* is true but *if A then (it is necessary that A)* is false when *A* is contingent. Byrne and Johnson's synonymy claim even fails in their own mental model theory of the modals (Johnson-Laird & Byrne, 2002). In terms of mental model theory, *it is necessary that (if A then A)* is true because every mental model of *A* is trivially a mental model of *A*. But it is false for contingent *A* that *if A then (it is necessary that A)*, which implies that *A* holds in all mental models even when *A* only represents a contingent fact.

Byrne and Johnson-Laird argue analogously (referring to Girotto & Johnson-Laird, 2010) that people will tend to interpret questions about the probability of a conditional as if *probability* also applies only to the main clause. It is certainly true that people appear to treat *What is the probability of if A then B?* and *If A then what is the probability of B?* as equivalent. However, note that the latter question has a scope ambiguity. To be unambiguous, what Byrne and Johnson-Laird (and Girotto & Johnson-Laird) are claiming is that people turn a wide scope question *What is the probability of (if A then B)?* into the narrow scope, "main clause" question *If A then (what is the probability of B)?* Yet Byrne and Johnson-Laird do not consider that these questions are "synonymous". They fallaciously hold that the definite answer to *If A then (what is the probability of B)?* is the conditional probability (see also Girotto & Johnson-Laird, 2010, p. 111), but that the correct answer to *What is the probability of (if A then B)?* is P(not-A or B), the probability of the material conditional (Byrne & Johnson-Laird, 2009, 2010). The *not-A* cases are not void in mental model theory, but are possibilities that make *if A then B* true, implying that they are "winning" outcomes for a bet on the conditional.

There are some important points to elaborate on in response (see also Evans & Over, 2010, and Over et al., 2010) to these, not fully consistent claims. People do not always interpret wide scope forms as narrow scope forms. They even interpret what might seem to be narrow scope uses as wide. More precisely, they often resolve a scope ambiguity with a wide and not a narrow reading. There is a standard example in philosophical logic: the quote from many sources, *All that glitters is not gold,* which has a classic scope ambiguity. If the *not* in it were actually read to have narrow scope, it would mean, *All that glitters is (non-gold)*, which is false. However, the ordinary interpretation of the quote comes from the wide scope reading of *not*, which means *Not (all that glitters is gold)*, which is true.

Conditionals containing modal terms, such as *If Bill is a bachelor then he must be unmarried* and *If Linda is anti-nuclear then she probably supports the Green Party*, also have scope ambiguities that are given wide scope readings by those who avoid modal fallacies (Edgington, 1995, p. 269). The fact that people utter scope ambiguities in “think aloud” studies of conditionals (Girotto & Johnson-Laird, 2006) is not evidence that they give these narrow scope interpretations. Indeed, that most people reply with the conditional probability, P(B|A), to the question with a scope ambiguity, *If A then what is the probability of B?*, is strong evidence that they interpret it as the wide scope question, *What is the probability of (if A then B)?*

To see the problem with narrow scope, “main clause” interpretations, suppose the scope of *probability* in *If A then what is the probability of B?* were narrow. To claim now, as Johnson-Laird and his collaborators do, that this narrow reading obviously asks for the conditional probability is to commit a modal fallacy. The conditional probability judgment only comes from the wide scope reading. Consider the simplest conditional probability judgment that we can make about all statements

A, P(A|A) = 1. This judgment corresponds to the wide scope reading: *we are certain that (if A then A)*. The narrow scope version, *if A then (we are certain that A)*, does not correspond to the trivial P(A|A) = 1, but to the delusion that one is omniscient. The wide scope interpretation is a judgment of absolute confidence in the epistemic relation between *A* and *A*, and that can be inferred for all *A* by anyone who can derive *A* from *A*. The narrow scope use is not synonymous with the wide scope use and P(A|A) = 1 for all *A*, but implies omniscience: that for all *A*, whenever A holds, we have absolute confidence in *A*.

It might be charged that our argument assumes a subjective interpretation of *probability* and *certainty*, but we need no such strong assumption. Let *A* be a contingent statement, and suppose that scientists cannot yet say whether *A* holds or not. That is, scientists agree that *it is not certain that A*. Let *A* be, say, that there was never primitive life on Mars. All scientists will also agree that P(A|A) = 1, but that cannot mean *if A then (it is certain that A)*. For by using *if A then (it is certain that A)* as the major premise and their agreed minor premise that *it is not certain that A,* scientists could learn by *Modus Tollens* that *not-A* holds, thereby contradicting the fact that they cannot yet say whether *A* holds or not. Scientists cannot infer that there was once primitive life on Mars from the triviality that P(A|A) = 1 and the well known fact that *A* is uncertain! This argument can be generalised to imply any number of absurd consequences, and thus, to repeat, conditional probability judgments come from wide scope interpretations. They are not equivalent to narrow readings that attach probability terms to the consequents of conditionals.

All Byrne and Johnson-Laird's (2009, 2010) modal fallacies come down to the charge that P(if A then B) = P(B|A) is "erroneous". What they think is "correct" is P(if

A then B) = P(not-A or B). Now our experimental materials and design were made as transparent as possible to forestall any claim that people are making "erroneous" judgments. We did not use conditionals with scope ambiguities in our materials. In our design, there was a clear objective ground for probability judgments. Previous studies simply stated the frequencies of the various Boolean cases: the number of *A & B*, *A & not-B*, *not-A & B*, and *not-A & not-B* instances. In contrast, our frequency information was visually displayed. There was no need to estimate the probability of a conditional or of its component parts independently. Computational difficulty was thus avoided, and no combinatorial analysis was necessary. When a computation was needed, it was extremely simple (as in taking the ratio of two small numbers).

The design will test theories of the natural language indicative conditional, *if A then B*. In probability theories, the indicative conditional is a probability conditional. These theories imply two hypotheses. First, that people will judge P(if A then B), to be the conditional probability, P(B|A), and interpret a bet on *if A then B* as a bet on a probability conditional. Second, people will judge that an *A & B* outcome makes the indicative conditional *true* and *wins* the conditional bet, that *A & not-B* outcome makes the indicative conditional *false* and *loses* the conditional bet, and that the *not-A* outcomes make the indicative conditional and the conditional bet *void*: not *true* or *false* and not *won* or *lost*. Against these probability theories, mental model theory states that the "correct" judgment about P(if A then B) is equal to P(not-A or B) and implies that a bet on *if A then B* is a bet on *not-A or B*, turning the *not-A* cases into winning outcomes. This theory also states that people will form an *initial* mental model *A & B* of *if A then B* and not always expand that to the *fully explicit* models of the material conditional, *not-A or B*. However, our transparent materials should ensure, if any materials do, that supposed mental models of the conditional are fully explicit.

Method

Material. We used a small number of chips in two colours, black and white, and in two shapes, round and square. For simplicity, the chips were displayed in two rows - see Figure 1 – for the participants. The questions asked were stated to be about a chip chosen at random from this display. The results of any computations could be given in the form of fractions, with numbers smaller than eight in the numerator and the denominator. The chips were represented on the first page of a booklet, with six questions to be answered in the booklet.

Insert Figure 1 about here

Participants. One hundred and seventy-eight participants were solicited in a public library. All of them had completed high school and were native speakers of French. Their background covered all disciplines from zero to ten years of higher education, with a mean of four years. They worked at their own pace.

Design. Participants were randomly allocated to six groups, which all answered six questions. Groups 1 and 2 answered questions about an indicative conditional, IC. Groups 3 to 6 answered questions about a conditional bet, CB. We will list the six questions in order and describe the different versions of these questions that the six groups answered.

Questions 1 and 2

Groups 1 and 2 received the indicative conditional IC, of the form *If A then B*, on Figure 1, *If the chip is square then it is black*. This conditional was described as a “sentence”, and these participants were asked in question 1, *What are the chances the sentence is true?* Question 2 for them was, *What are the chances the sentence is false?*

Groups 3 and 4 received the conditional bet CB on Figure 1. Mary says to Peter, *I bet you 1 Euro that if the chip is square then it is black*. The questions for these groups paralleled those given to groups 1 and 2, except these questions were about Mary’s winning, or losing, her bet. Question 1 was, *What are the chances that Mary wins her bet?*, and question 2 was, *What are the chances that Mary loses her bet?*

Groups 5 and 6 were CB groups like groups 3 and 4 and were given the same conditional bet and the same questions 1 and 2 that groups 3 and 4 were. Groups 5 and 6 differed from groups 3 and 4 at questions 3 and 4.

The parallel questions for the IC groups and the CB groups were designed to test whether the participants would make parallel judgments of P(B|A) for the probability of truth and of winning questions and of P(not-B|A) for the probability of falsity and of losing questions. These answers are implied by probability theories of the conditional.

Questions 3 and 4

Groups 1 and 2 were given different versions of questions 3 and 4 about the two false antecedent, *not-A* cases.

Group 1 had a two-option response for question 3. They were asked to suppose that the chosen chip is round and black, of the form *not-A & B*. They were then asked, *Do you think that the sentence is true or false?* This two option question served as a control to find out how the participants would respond in the absence of a third, void option.

Group 2 had a three-option response for question 3, which now explicitly included a void option. They were asked to suppose that the chosen chip is round and black, of the form *not-A & B*. They were then asked, *Do you think that the sentence is true, false, or neither true nor false?*

Group 1 again had a two-option response, and group 2 a three-option response for question 4, but this time they were asked to suppose the other *not-A* case, that the chip was round and white, of the form *not-A & not-B*. Question 4 was otherwise like question 3.

Group 3 had a two-option response for question 3, under the supposition that the chip was round and black, *not-A & B*, *Do you think that Mary wins or loses her bet?* Group 4 had a three-option response for question 3 under the same supposition, *Do you think that Mary wins her bet, loses her bet, or nobody wins: the bet is called off?* Group 3 had the same two-option response, and group 4 the same three-option response, for question 4, but now supposing that the chip was round and white, *not-A & not-B*.

The probability theories of the conditional imply that the participants would select the void option - neither true nor false and neither winning or losing - at least when it was explicitly offered to them, as the answer to the questions about the *not-A* outcomes.

Groups 5 and 6 were CB groups like groups 3 and 4 and were given the same conditional bet. But groups 5 and 6 differed from groups 3 and 4 at questions 3 and 4. Instead, of being asked about the *not-A* outcomes, groups 5 and 6 were asked to make judgments under either the supposition that the chosen chip was black, a *B* outcome, or that is was white, a *not-B* outcome. Group 5 was asked in question 3 for the conditional probability that Mary wins her bet given that the chip is black, P(w|B). They were then asked in question 4 for the conditional probability that she loses her bet given that the chip is black, P(l|B). Group 6 was asked in question 3 for the conditional probability that Mary wins her bet given that the chip is white, P(w|not-B). It was then asked in question 4 for the conditional probability that she loses her bet given that the chip is white, P(l|not-B).

For questions 3 and 4, participants were also given the opportunity to write a justification of their responses.

Questions 5 and 6

The last two questions, questions 5 and 6, were given to all six groups and in a counterbalanced order. The aim was to test whether the participants could distinguish a request to evaluate the probability of the conjunction, P(A & B), from a request to evaluate the conditional probability, P(B|A). To avoid a possible effect of training or familiarization, a new display was used, showing one row of four round chips (one black, three white) and one row of three square chips (one black, two white), and the questions were still about a chip chosen at random. Question 5 asked for P(A & B), *What are the chances that the chip is square and white?*, and question 6 asked for P(B|A), *Suppose the chip is square. What are the chances that it is white?*

We will now summarize the questions put to the six groups. The two indicative conditional IC groups, groups 1 and 2, were identical except for the third and fourth questions, where group 1 had the two-option response format and group 2 had the three-option format. The four conditional bet, CB groups also differed from one another only at the third and fourth questions, where group 3 has the two-option format, group 4 had the three-option format to answer the questions about the chances of winning or losing given the *not-A & B* and the *not-A & not-B* outcomes, respectively, and groups 5 and 6 had the questions about the chances of winning or losing given the *B* outcome and the *not-B* outcome, respectively. The composition of the questions for each group is presented in Table 1.

Insert Table 1 about here

Results

Questions 1 and 2. These questions asked for the probability that the sentence is true/false, or that Mary wins/loses her bet. Participants were highly coherent, that is, they almost always respected the additivity of the two probabilities; the results are presented for both questions paired together in Table 2.

Insert Table 2 about here

It is apparent that the distribution for the indicative conditional, IC, and the conditional bet, CB, are nearly identical in the way that they parallel each other.

Next, it is by far the conditional probability, P(B|A), that was chosen (almost two thirds of the cases), followed by the conjunctive probability, P(A & B) in about 16% of the cases. The probability of the material conditional, P(not-A or B), occurred in only 2% of the cases. These results extend previous research on the IC to the CB and provide further confirmation of probability theories of the conditional, which imply that the IC and CB are similarly related to the conditional probability, P(B|A).

There were about 18% of answers to these first questions that did not coincide with any of the three major types of response. They showed a great variability: two of them occurred twice, the others only once and all were clearly erratic. Some summed up to 1 and were given as percentages (e. g. 1%, 99%) or fractions (e. g. 4/5, 1/5) but the majority did not sum up to 1 (e. g. 50%, 10% or 1/3, 1/7).

Questions 3 and 4, Groups 1 to 4. In questions 3 and 4 for groups 1 to 4, the drawn chip is assumed to be a *not-A & B* or a *not-A & not-B* case. Most participants gave the same answer to these two questions. Table 3 indicates the distributions for the IC and the CB conditions and for the 2- and 3-choice formats.

Insert Table 3 about here

For the three-choice format, comparison of the IC and CB conditions shows similar distributions, that is, if the response categories are rank-ordered, the same order obtains, with the *neither..nor* option (resp. *void* option) ranking first by far with impressively high rates, followed by the false (resp. *lose*) option. The only notable difference (which was significant at the .05 level) is the relatively higher frequency of *void* answers (79%) as compared to the *neither..nor* answers (52%).

For the two-choice format, we first mention a remarkable observation. For the two-option groups, a sizeable proportion of participants did not choose any of the two options (or, less often, chose both) even though this was not explicitly permitted, and wrote an explanation for this. We quote two typical examples: "Because the chip is round it is not concerned by the phrase 'if the chip is square'". "Neither win nor lose because the bet holds only if the chip is square; here there is no bet".

The *false* option and its counterpart, the *lose* option, were the most frequently chosen in the two-choice format (47% and 40% respectively). However, the similarity between the two distributions did not extend to the other options. Indeed, the *true* option was the next most frequently chosen whereas its counterpart - the *win* option - was the least often chosen, and the two distributions differed at the .05 level of significance (chi-square = 7.68, df = 2). The participants were even more disinclined to declare the void, *not-A* case as one of winning for Mary than they were to identify it as making the indicative conditional true. This appears natural if we recall our point above that the bet has a positive expected value of 29 cents for Mary, and she “loses” that value when the bet is void. Correlatively, they were more hesitant to decide between *win* and *lose* than they were to decide between *true* and *false*. This is shown by their relative inconsistency from the third to the fourth question (20%) and by the greater frequency of spontaneous *neither..nor* answers in the CB condition than in the IC condition (27% vs 13%). Though these figures are not significantly different, a similar trend was observed in a replication study that will be described later.

Questions 3 and 4, Groups 5 and 6. In questions 3 and 4 for groups 5 and 6, in the CB condition, the drawn chip is assumed to be black *B* (group 5) or *not-B* (group 6), and the participants evaluate the probability of winning or of losing under that assumption. Table 4 shows the responses.

Insert Table 4 about here

There was a great variety of answers suggesting that participants were indeed confused by these questions. No one explanation accounts for the responses. Assuming the probability is given by the material conditional fares very poorly with between 0% and 20% of responses; assuming it is given by the conjunction is a better predictor with between 27% and 59% of responses; and assuming it is given by the conditional probability lies in between with from 0% to 47% of responses. These responses may have been the result of making different presuppositions about the void, *not-A* case, or thinking of this as one in which Mary “loses” her positive expected value for the bet.

Questions 5 and 6 for all groups. There was a new display of seven chips to test whether the participants could correctly answer questions about the probability of a conjunction and a conditional probability. Three chips were square, with one black and two white. Question 5 asked for the conjunctive probability, P(white & square). For all the conditions pooled together, there were 85% correct responses (2/7). Among the other responses, 6% indicated 2/3 or 2/5 (presumably a confusion with a conditional probability), 4% indicated 5/7 or 3/7 (presumably the probability of only one of the two conjuncts), and the remaining 5% were various erratic values. Question 6 asked for the conditional probability, P(white|square). Again for all the conditions pooled together, there were 82% correct responses (2/3). The most frequent erroneous response was the probability of the conjunction (2/7), found in 10% of the cases. There were 3% of 1/3 responses (the complementary probability, an apparent processing error), 2% of 2/5 responses (the inverse conditional probability), and the remaining errors (3%) were erratic values. In brief, with our materials requiring only simple counting and giving a ratio, the vast majority of participants correctly gave the probability of the conjunction, P(A & B), and the conditional probability, P(B|A). The confusion between these did not exceed 10% of the cases.

Discussion

There was general support for probability theories of the conditional grounded in the work of Ramsey and de Finetti. These theories imply that there is a close relation between the indicative conditional IC, the conditional bet CB, and the conditional probability. There were few differences between the response rates to the IC and CB conditions. First, the distributions over the major categories of response for the probability of the truth of the conditional and the probability of winning the conditional bet (CP, conjunction, MC) were parallel to each other and almost identical, as seen in Table 2. Second, the distributions of answers to questions about the truth and falsity, or winning and losing outcomes, differed significantly in only two cases. These were *not-A* outcomes in which the antecedent was false. In the two-choice format, there were slightly more *true* judgments (37%) for the IC than *win* judgments (13%) for the CP, and in the three-choice format, the IC condition elicited fewer *neither* judgments (52%) than the CB condition (79%).

There was disconfirmation of the mental model theory of conditionals. This theory implies that people will represent a conditional *if A then B* either in an initial model of the conjunction *A & B* or in fully explicit models of the material conditional *not-A or B*. There were few conjunctive responses, and material conditional responses were negligible in our fully transparent materials. Given *not-A* in two-options conditions, in which the participants had to choose whether IC was true or false, or CB won or lost, the modal response was to judge that IC is "false" and CB "lost". Mental model theory implies that they should have opted for "true" and "won". The implication of mental model theory that a bet on the indicative conditional is a bet on the material conditional is not at all supported.

The false antecedent, *not-A* outcomes are of great theoretical interest. Mental model theory implies, against probability theories, that these are true and winning possibilities, in which the indicative conditional is true and the conditional bet is won. Mental model theory cannot explain our results, summarized in Table 3, that more people prefer to call these false and losing possibilities in the two-choice conditions, and that most people judge these to be void - neither true nor false/neither winning nor losing - outcomes in the three-choice conditions.

Probability theories of the conditional directly predict people's judgments in the three-choice conditions. The void - "neither true nor false" and "neither winning nor losing" - option is made explicitly available in these conditions. Given *not-A*, this is the option that people should select according to probability theories, and the option they do tend to choose. The high rate of these void responses supports the position, based on the de Finetti table, that people use a three-outcome classification for IC and CB conditionals: *true/win*, *false/lose*, or *void*.

Probability theories could also account for the differences between the IC and the CB groups in the two-choice conditions, with the help of a closer look at Table 3. Given *not-A* in the two-choice CB condition, if the group 3 participants who are fluctuating between the responses "win" and "lose" (20%, third row of the table) are pooled with those who respond "win" consistently (13%, first row), the rate becomes close to its IC counterpart (37+3=40%). This suggests an even greater reluctance in the CB conditions to judge *not-A* as a “winning” outcome than there is in the IC conditions to judge it as a “true” possibility, and that makes sense pragmatically. A bet is goal orientated: people's aim in betting is to get a monetary or other benefit. From this point of view, the *not-A* void case can be seen as a disappointment, the “loss” of an opportunity to get the benefit. This is pronounced in our materials by the fact that the bet has a positive expectation for Mary, and that is hardly compatible with an evaluation of the void case as a “win” for her. It should be possible, however, to make the IC conditions more practical or utilitarian in this way. Putting an indicative conditional into the context of an argument between two people, with a benefit attached to winning the argument (if only personal satisfaction), should make judgments about the indicative conditional even more like judgments about a conditional bet.

Questions specific to groups 5 and 6 in the CB condition received intriguing responses. Group 5 was asked about the chances of Mary’s winning/losing her conditional bet given that the consequent of the conditional holds, *B*. Group 6 was asked about the chances of Mary’s winning/losing her conditional bet given that the consequent of the conditional does not hold, *not-B*.

There was evidence of some confusion about these seemingly quite difficult questions. When B is supposed, the modal response (55%) was that the chances that Mary will win is P(A|B), and the modal response (59%) that she will lose is P(not-A|B). It is noteworthy that no participant gave 100% as her chance of winning. The answer of 100% for her chance of winning would be expected if the participants also supposed that the void, *not-A* outcomes were to be ignored, leaving only *A & B* outcomes. It will be recalled that, in answering question 1, most participants did make the supposition that *not-A* outcomes were irrelevant and to be ignored. Question 1 was simply about Mary's chances of winning the conditional bet, and most participants answered that with P(B|A). They interpreted the question as about the number of *A & B* cases out of *A* cases, which comes from supposing *A*. It would appear that supposing *B* precludes a supposition of *A* as well. Supporting this point is that no one gave 0% as Mary's chance of losing under the supposition of *B*. That would be the answer if *A* was supposed to hold as well.

When *not-B* is supposed to be the case, the results were even less clear-cut. On the chances that Mary will win, the modal response (47%) was understandably 0%. However, on the chances she will lose, the modal response (27%) was 100%, and the conditional probability of *A* given *not-B*, P(A|not-B), was given by only 20% of participants.

This pattern of responses for groups 5 and 6 could be explained in more than one way. Consider first the *B* case. It may be that people are generally polarized towards winning, and they thought of the void case, *not-A & B*, as a failure to fulfill this goal of winning and effectively as a disappointing "losing" outcome. That would explain why many (59%) judged the chances of Mary's losing as P(not-A|B). Alternatively, it may be that people, more specifically, grasped that Mary had the advantage in the bet, which had a positive expected value for her, and that she "lost" this expected value in the *not-A & B* outcome. This explanation again implies that people would take the chances of Mary's "losing" as P(not-A|B). Both explanations also imply, of course, that Mary's chance of winning is P(A|B), and indeed 48% of participants gave both P(A|B) and P(not-A|B) as the chances of winning and losing, respectively.

Under the *not-B* supposition, the majority of participants (47%) said that Mary's chances of winning are null, and 20% that her chances of losing are equal to P(A|not-B). However, slightly more (27%) estimated that she was certain to lose, in effect taking both *A & not*-B and *not-A & not-B* as “losing” outcomes. The latter could have counted as “losing” either for the general reason (it is a failure to win) or the specific one (Mary “loses” her expected value) we have just given. Recall that, given only a two-choice option, people tend to classify the *not-A & B* and *not-A & not-B* outcomes as ones in which Mary “loses” her bet. In sum, we can find some coherence in the data if the void outcomes, *not-A & B* and *not-A & not-B*, are thought of as “losses” (for either the general or specific reason) in some sense over and above *A & not-B* as the most basic losing outcome. This hypothesis about the void outcomes will have to be investigated in future work.

Consider next the conjunctive response, P(A & B), to the questions about the chances that the indicative conditional is true (group 1) and that Mary wins her bet (group 3). This response corresponds to taking the number of *A & B* cases out of all outcomes, not only the *A* ones but the void *not-A* ones as well. Probability theories of the conditional, based on the Ramsey test and the de Finetti table, imply that it is the former, the *A & B* cases out of *A* outcomes, yielding the conditional probability, P(B|A), that is fundamental. It is these theories that imply that the *not-A* outcomes make the conditional utterance or conditional bet void. No epistemic standing, or actual money in a bet, is won or lost in these void outcomes, and it is P(B|A) which determines who has the advantage in the bet or whether it is fair. The majority response is P(B|A), but the question is why do some participants have the P(A & B) response? A possible answer is that the P(A & B) response takes fewer mental operations. The P(B|A) response is the result of one mental operation of supposing

A, that the chip is square, and then another to take the ratio of black squares *A & B* to all the squares *A*. However, under a cognitive load, or with lower cognitive abilities, it is easier to bypass the suppositional *A* step and look only at the ratio of black squares *A & B* over all of the chips, leads to the P(A & B) response.

Probability theories of conditionals imply that there is a close parallel relation between indicative conditionals, conditional bets, and conditional probability, and that is what we have found, as shown in Table 2. There is not an absolutely perfect match in people's judgments about the truth of indicative conditionals and the winning of conditional bets, but that cannot be expected when the ordinary terms *true* and *win* are used. There are many uses of these terms in natural language, and as any standard dictionary will show, *true* is even less univocal than *win*. People do not necessarily acquire, or give up, anything concrete when they "win", or "lose", as when they "win" an argument. They do not always have to use *true* to refer to an objective state of affairs, like an *A & B* outcome or such an outcome out of *A* possibilities, but can use it purely subjectively to indicate agreement with a point of view, even one of individual taste. For example, we may say "true" in response to someone who claims, "You should use red rather than white wine if you make coq au vin", when other people (lovers of coq au Riesling) would say "false", though we all agree there is no objective fact of the matter. There is a pleonastic or pragmatic use of *true* (Adams, 1998; Edgington, 2003) that expresses some kind of endorsement. Nevertheless, our experiment broadly confirms two hypotheses implied by probability theories of the conditional. Both indicative conditionals and conditional bets are closely related to the conditional probability, and the classification of *true*, *false*, and *void* for indicative conditionals parallels that of *win*, *lose*, and *void* for conditional bets.

Replication of the study

The above experiment was replicated with a sample from a population of a lower academic level, 234 undergraduate psychology students. All the trends that have been mentioned were supported, showing that our results are robust. Interestingly, there was no difference between the higher and the lower academic samples for any question in the conditional bet conditions. For the indicative conditional, there were differences on three questions, always in the direction of a higher performance for the higher academic level sample. For the chances that the sentence is true/false, the conditional probability responses were 73% and 69% for the higher sample versus 52% and 50% for the lower one ($p<.01$ and $p<.05$, respectively). This difference was entirely due to a higher proportion of P(A & B) conjunctive responses in the lower academic sample. For question 5 about P(A & B), the percentage correct was 88% for the higher sample versus 61% for the lower one ($p<.001$). It is interesting that question 6 about the conditional probability did not yield any difference between the two samples, as though this were the better entrenched concept. These results support the findings of Evans et al. (2007) that participants of relatively low cognitive ability tend to have more conjunctive interpretations of the probability of an indicative conditional, and those of relatively higher cognitive ability tend to have the conditional probability interpretation (see also Evans, Handley, Neilens, & Over, 2008, and Oberauer, Geiger, Fischer, & Weidenfeld, 2007). It is of interest that there was no such effect for the conditional bet.

There is evidence that people of higher and lower cognitive capacity can be alike in judging conditionals that are directly related to benefits and costs (Stanovich & West, 2000). As we have pointed out, conditional bets aim at getting a benefit, and indicative conditionals can be given more of this spin (in an argument in our example). The effect of expected benefits and costs on indicative conditionals will have to be explored in future work, but any such effect could be explained by a probabilistic or Bayesian approach to the study of reasoning (Evans & Over, 2004; Oaksford & Chater, 2007). Overall, our studies support the new probabilistic paradigm in the psychology of reasoning and its implication: indicative conditionals, conditional bets, and conditional probability are closely related to each other.

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Acknowledgements

We would like to thank Thomas Charreau, Angelo Gilio, Giuseppe Sanfilippo, and the participants of the Toulouse workshop on reasoning, March 29-30, 2010, for helpful comments.

Figure 1. The display of the chips and the conditional sentences in the indicative conditional and the conditional bet conditions (for questions 1 to 4).

This drawing represents chips.

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■ ■ ■ □

Indicative conditional conditions (IC):

A chip is chosen at random.

Consider the following sentence:

If the chip is square then it is black

Conditional bet conditions (CB):

A random chip is to be drawn in a fair way

Mary tells Peter:

"I bet you 1 Euro that if the chip is square then it is black"

Peter replys: *"I take it up"*

Each of them puts one Euro on the table. They agree that the winner will pocket all the stake.

Table 1. A summary of the questions that define the six experimental groups.

Notations:

IC = the indicative conditional; CB = the conditional bet; A = the antecedent of IC or CB: B = the consequent of IC or CB; P = probability; t = true; f = false; w = win; l = lose; V = the evaluation (t or f, w or l, or neither) of IC or CB for various outcomes.

	Questions					
	1	2	3	4	5	6
Groups						
IC: Group 1 (2-option) and Group 2 (3-option)	P(t)	P(f)	V(-A&B)	V(-A&-B)	P(A&B)	P(B|A)
CB: Group 3 (2-option) and Group 4 (3-option)	P(w)	P(l)	V(-A&B)	V(-A&-B)	P(A&B)	P(B|A)
CB: Group 5	P(w)	P(l)	P(w|B)	P(l|B)	P(A&B)	P(B|A)
CB: Group 6	P(w)	P(l)	P(w|-B)	P(l|-B)	P(A&B)	P(B|A)

Table 2. Percentage of joint answers to the first and second questions.

	Indicative conditional	Conditional bet	average (IC and CB)
	What are the chances that		
Question 1:	the sentence is true	Mary wins her bet	
Question 2:	the sentence is false	Mary loses her bet	
Conditional Probability			
3/4 for Question 1 and 1/4 for Question 2	69	61	63.7
Conjunction			
3/7 for Question 1 and 4/7 for Q2	14	17	16
Material Conditional			
6/7 for Question 1 and 1/7 for Question 2	2	2	2
Other	15	20	18.3

Table 3. Distribution of the answers in percent to the third and fourth question for groups 1 to 4.

	Indicative Conditional			Conditional Bet	
	group 1 (2-choice)	group 2 (3-choice)		group 3 (2-choice)	group 4 (3-choice)
answer					
true for both questions	37	7	*win* for both questions	13	7
false for both questions	47	28	*lose* for both questions	40	10
true for one *false* for the other	3	3	*win* for one *lose* for the other	20	3
neither true nor false for both questions	13*	52	*void* for both questions	27*	79
mixed °	0	10			

The target sentence is: *If the chip is square then it is black*

Indicative conditional

Q3. Suppose the chip is **round** and **black**. Do you think the sentence is:

true false neither true nor false [group 2 only]

Q4. Suppose the chip is **round** and **white**. Do you think the sentence is:

true false neither true nor false [group 2 only]

Conditional bet

Q3. Suppose the chip is **round** and **black**. Do you think that

Mary wins her bet Mary loses her bet the bet is called off [group 4 only]

Q4. Suppose the chip is **round** and **white**. Do you think that

Mary wins her bet Mary loses her bet the bet is called off [group 4 only]

° mixed = *neither..nor* for one question and *false* for the other

* these are spontaneous expressions of the third option

Table 4. Distribution of the answers in percent to the third and fourth questions for groups 5 and 6. Left column: participants' answers; right column: percentage choosing each answer.

The target sentence is: *If the chip is square then it is black*

Group 5

Suppose the chip is **black**. What are the chances that Mary **wins** her bet?

1 (or 100%)		0%
3/5	= P(A|B)	55%
3/4	= P(B|A)	18%
5/7	= P(B)	10%
other		18%

Suppose the chip is **black**. What are the chances that Mary **loses** her bet?

0 (or 0%)		7%
2/5	= P(-A|B)	59%
3/4	= P(B|A)	7%
1/4	= P(-B|A)	7%
other		20%
3/5 on Q3, 2/5 on Q4		48%

Group 6

Suppose the chip is **white**. What are the chances that Mary **wins** her bet?

0 (or 0%)		47%
1/2	= P(A|-B)	17%
2/7	= P(-B)	10%
1/4	= P(-B|A)	10%
other		16%

Suppose the chip is **white**. What are the chances that Mary **loses** her bet?

1 (or 100%)		27%
1/2	= P(-A|-B)	20%
0%		13%
3/4	= P(B|A)	10%
1/4	= P(-B|A)	10%
other		20%
1/2 on Q3, 1/2 on Q4		17%
0% on Q3, 100% on Q4		27%