The Semantics of Paranumerals

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Abstract
The paper contrasts two semantic subclasses among expressions combining with numerals. These subclasses, exemplified respectively by *at least* and *more than,* are further contrasted with bare numerals. Even if they often make the same contribution to truth conditions, there are grounds for distinguishing numerals, 'numerical comparatives' (*more than)*, and 'set comparators' (*at least*) on the basis of dynamic properties, to do especially with anaphora and apposition. The paper makes the following claims: (i) bare numerals introduce into the representation a set of exactly n individuals; (ii) numerical comparatives (*more/less than n)* only introduce the maximal set of individuals Σx satisfying the conjunction of the NP and VP constraints, and compare the cardinality of this set to n; set comparators (*at least/at most*) introduce two sets into the representation: Σx, and a witness set, the existence of which is asserted, which is constrained as a set of n Xs, X being the descriptive content of the NP. The paper is presented in the framework of Discourse Representation Theory and is based on French data.

1. Introduction[*]

Besides numerals (*one, two, three,…*), there is a set of expressions which combine easily, if not exclusively, with numerals: *at least, at most, exactly, more than, less than*. Let us call these expressions *paranumerals;* this term is purely descriptive and does not contain any implicit analysis of these expressions, for which we cannot rely on any accepted terminology. Similar expressions exist in many languages and, for the sake of illustration, I will use French data including: *au moins* ('at least')*, au plus* ('at most')*, exactement* ('exactly')*, plus de* ('more than')*, moins de* ('less than') *en tout* ('in all').

Paranumerals can combine with numerals, but, as a rule, they do not combine with quantifiers and indefinites: *plus de deux* ('more than two')*, moins de trois* ('less than three')*, exactement quatre* ('exactly four'), **plus de plusieurs* ('more than several')*, *au moins peu* ('at least few'). There are some indefinites with respect to which the behaviour of paranumerals is not regular: **exactement quelques* ('exactly some')*, plus de quelques* ('more than some')*, au moins quelques* ('at least some')*, au plus quelques* ('at most some'). As for definites, some paranumerals combine with them, others do not:[1]

(1)   a. *J'ai invité au moins Pierre et Jean.*
              'I invited at least Pierre and Jean.'
        b. *J'ai invité Pierre et Jean au plus.*
              'I invited at most Pierre and Jean.'
        c. *Au moins Pierre et Jean sont venus.*

'At least Pierre and Jean came.'
   d.   *Plus que Pierre et Jean sont venus.
        'More than Pierre and Jean came.'
   e. *Moins de Pierre et Jean sont venus.
        'Less than Pierre and Jean came.'

The semantic literature, in particular the algebraic approaches of Generalized Quantifier Theory (Barwise & Cooper 1981) and Boolean semantics (Keenan & Stavi 1986), generally tends to take all paranumerals to behave similarly and to see them as close to numerals. This view is based on the following list of truth-conditional equivalences:

un ('one')	↔	*au moins un* ('at least one')
plus de n ('more than n')	↔	*au moins n+1* ('at least n +1')
n exactement ('n exactly')	↔	*n en tout* ('n in all') ↔ *n au plus* ('at most n'), *n au moins* ('at least n')

In this paper, I will try to substantiate the following claims:
(i) The semantics of bare numerals and numerals+paranumerals should be sharply contrasted; I shall focus in particular on the difference between *n* and *at least n.*
(ii) There are at least two different kinds of paranumerals, exemplified respectively by *plus de* ('more than') and *au moins* ('at least').
(iii) It is difficult to say for certain if *exactement* ('exactly') is of the *more than* kind or of the *at least* kind, but it can be established that *en tout* ('in all') and *exactement* ('exactly') behave differently.
   The first two claims will be discussed in detail; as for the third claim, I will provide only an outline of the discussion.
   The general perspective adopted here combines truth-conditional semantics (which tends to treat all paranumerals in the same way) and an in-depth exploration of their dynamic properties, based on two kinds of data:
A. The interpretation of definite anaphors for these expressions, an exploration initiated by Kadmon (1987) and illustrated by examples like (2):

   (2)   *Pierre invitera au moins deux personnes. Il les recevra dans l'entrée.*
         'Pierre will invite at least two people. He will receive them in the hall.'

In such examples the problem is to determine how the pronoun is interpreted: as referring to a set of exactly two persons or as referring to the maximal set of invited persons.
B. The interpretation of appositives, illustrated by examples like (3):

   (3)   *Pierre invitera au plus deux personnes: son père et sa mère.*
         'Pierre will invite at most two people: his father and his mother.'

To some extent, apposition data can be used to elucidate the problem raised in (A): it seems that appositives are often interpreted as enumerating the set introduced by the expression they are appended to.
   I will argue that in order to understand correctly the semantics of expressions of the form *(paranumeral) n AB*, one must distinguish two sets:
- the maximal set of individuals satisfying the intersection: $E_{max} = A \cap B$;

- a set of exactly *n* members: E_n.

More precisely, the claim is that what distinguishes these expressions is the nature of the set(s) relevant for computing their representation. Those sets are given in the following table, in which we introduce a working terminology for the different categories we wish to distinguish:

expression	*relevant set(s)*	*example*
numerals	E_n	three boys
numerical comparatives	E_{max}	more than three boys
set comparators :	E_n, E_{max}	at least two boys

The paper gives empirical arguments based on the dynamic properties of these expressions, and proposes a semantics based on the relevant sets which is formulated in the DRT framework (Kamp & Reyle 1993). The paper is grounded on the 'two-set' analysis introduced in Corblin (to appear) for expressions in the *at least* paradigm (set comparators). In order to keep the focus on a contrastive approach, I will not repeat here some details, discussions and references that the interested reader may find in that paper.

2. The Semantics of (Bare) Numerals: A Quick Overview

The classical truth-conditional analysis[2] of sentences containing a bare numeral *n* in a structure *n AB* is as follows:

(4) $[[n \ AB]] = 1$ iff $|A \cap B| \geq n$

It holds that the *n AB* sentence is true if and only if the cardinal of the intersection of A and B is at least *n*. It is, in general, supplemented by the classical pragmatic Gricean *implicature* – which does not hold for *at least n* sentences – that the speaker has no evidence that $|A \cap B| > 1$.

(5) $n = |A \cap B|$ From (3) and the Gricean implicature.

The approach treating bare numerals as indefinites adopted by Discourse Representation Theory admits (4), is agnostic about (5), but holds that the statement *n AB* 'introduces' for the following discourse a set of exactly *n* members, $E_n \subseteq E_{max}$. Consider for instance (6):

(6) *Deux étudiants ont appelé.*
     'Two students called.'

Sentence (6) is considered true if more than two did call, but it cannot be followed by (7):

(7) *# Ces trois étudiants étaient Pierre, Jean et Nicole.*
     'These three students were Pierre, Jean and Nicole.'

The ingredients of the solution for accommodating these data are represented in (8):

(8)

X student (X) $\lvert X \rvert = 2$ called (X)	Truthful embedding iff $E_{max} = \lvert S \cap C \rvert \geq 2$ Accessible for anaphora, E_n: $E_n \subseteq E_{max}$, $\lvert E_n \rvert = 2$ with S for students, C for callers.

This amounts to defining the truth of (8) by means of a truthful embedding, and to taking anaphora as a clue about what is made accessible by the sentence for later reference.

The kind of dynamic data exemplified by (9)

(9)   *Deux étudiants ont appelé... Ces deux (#trois) étudiants…*
        'Two students called…   These two (#three) students….

can be interpreted roughly as follows: the demonstrative NP *These n students* must be identified to a previously introduced discourse referent (DR), and its descriptive content *n students* must be satisfied by this DR. So, if the descriptive content of the demonstrative is 'exactly n', then its antecedent DR must be exactly *n.* That *These n Ns* means *exactly n Ns* seems to be established by the falsity or unacceptability of (10) and by the fact that (11) is a tautology:

(10)   **Ces deux étudiants sont Pierre, Jean et Nicole.*
        'These two students are Pierre, Jean and Nicole.'

(11)   *Ces n Ns sont   {a, …}$_E$ $\rightarrow$ $\lvert E \rvert = n$*
        'These n Ns are {a, …}$_E$ $\rightarrow$ $\lvert E \rvert = n$'

It must be concluded that *These n Ns* refers to exactly *n* Ns*,* which establishes that *n Ns* introduces exactly *n* Ns.[3]

Within the DRT approach, it is thus the (bare) numeral itself which introduces a set of exactly *n* Ns. This is a very important difference with respect to Kadmon (1987) and Evans (1980), who make the definite NP responsible for the unique (or maximal) interpretation of the anaphoric definite NP.

The apposition data seems to show that it is actually the numeral which is relevant. In sentences where a list of proper names is appended to an *n As* NP, lists of more than *n* names are not acceptable. As for sentences with a list of less than *n* names, they are not acceptable either, except with prosodic marking indicating clearly that the enumeration is not exhaustive.

(12)   *Deux personnes sont venues : *Pierre, Jean et André.*
        'Two people came: Pierre, Jean and André.'

(13)   *Est-ce que deux personnes sont venues? Oui. *Pierre, Jean et André.*
        'Did two people come? Yes: Pierre, Jean and André.'

In other words, anaphora and apposition show that the semantics of bare numerals in simple episodic sentences is such that *n AB* introduces a set of exactly *n* members satisfying A and B.

3. Numerical Comparatives: *plus de* ('more than'), *moins de* ('less than')

We use, as a working terminology, the label 'numerical comparatives' for the French equivalent of *more than n, less than n.*

    (14)   *Plus de cinq personnes sont venues.*
           'More than five people came.'

    (15)   *Moins de vingt étudiants se sont inscrits.*
           'Less than twenty students registered.'

According to the classical view, an expression like *more than n* is true in the same models as the numerical expression *n+1*.

    (16)   *J'ai écrit trois articles.*   ↔ *J'ai écrit plus de deux articles.*
           'I wrote three papers.'   ↔ 'I wrote more than two papers.'

This sounds roughly correct, at least if one operates only with integers on the considered domain. If, for instance, one is allowed to consider fractions, the equivalence does not hold, as exemplified by (17):

    (17)   *Cela mesure trois kilomètres.* ≠ *Cela mesure plus de deux kilomètres.*
           'It is three kilometers long.'   ≠ 'It is more than two kilometers long.'

Since one could have in mind, say, two and a half kilometers, it is not true that being more than two kilometers long implies being three kilometers long.

    But the semantics of numerals and paranumerals are different. It can be shown that numerical comparatives do not introduce a set of exactly *n* members, as numerals do (see section 2 above). Consider, for instance, the contrast between (18) and (19):

    (18)   *Deux personnes ont été contactées: Jean et Nicole.*
           'Two people have been contacted: Jean and Nicole.'

    (19)   *Plus de deux personnes ont été contactées: *Jean et Nicole.*
           'More than two people have been contacted: Jean and Nicole.'

Sentence (18) is fine for everyone, but most speakers feel that (19) is awkward.

    The contrast highlighted by apposition is confirmed by anaphora: (20) is correct, but (21) is not:

    (20)   *J'ai lu trois articles. Ces trois articles sont A, B et C.*
           'I read three articles. These three articles are A, B, and C.'

(21)   *J'ai lu plus de trois articles. *Ces trois articles sont A, B et C.*
        'I read more than three articles. These three articles are A, B, and C.'

In the light of the interpretation of these facts adopted before, we can conclude that numerical comparatives do not introduce any set E_n of cardinality *(exactly) n.* Moreover, there are data indicating that numerical comparatives do introduce the maximal set E_{max} into the semantic representation. The following discourses are perceived as natural by many speakers:

(22)   *J'ai cité plus de deux auteurs: Platon, Aristote et Sénèque.*
        'I mentioned more than two authors: Plato, Aristotle and Seneca.'

(23)   *J'ai cité Chomsky plus d'une fois: dans l'introduction, dans le chapitre 1,*
        'I mentioned Chomsky more than once: once in the introduction, once in
        chapter 1,...'

In (22), with conclusive intonation and the presence of *et* ('and'), the list is interpreted as exhaustive. In (23), it is only required that the list be of cardinality *n+1.*

It is fair to say that some speakers do not like sentences like (22), but anaphora confirms that E_{max} is actually part of the picture:[4]

(24)   *Elle a reçu plus de dix lettres, les a lues et classées.*
        'She received more that ten letters; she read and filed them.'

All the speakers consulted said that in (24) she read and filed all the letters she received. The same is true for (25), an example involving *moins de* ('less than'):
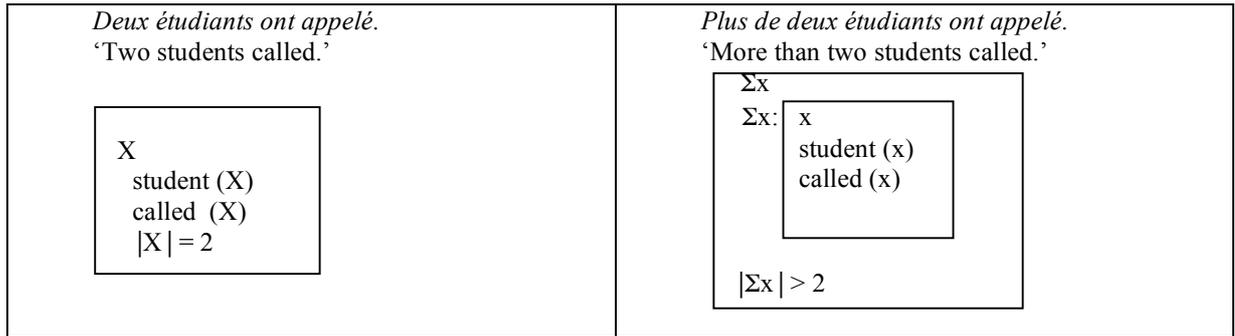
(25)   *Il a fait moins de cinq fautes. Il les a corrigées.*
        'He made less than five mistakes. He corrected them.'
        = He corrected all the mistakes he made.

We can conclude that the only set involved in the representation of numerical comparatives is E_{max}.

The fact that E_{max} is not relevant for numerals (see above) but is relevant for numerical comparatives is a confirmation (contra Evans) that it is the antecedent expression itself, not the definite anaphoric NP, which is responsible for the maximal interpretation. This is borne out by the similar behavior of anaphora and apposition (in which there is no definite NP to be interpreted).

In order to represent the maximal set, I will make use of the abstraction operator of Kamp & Reyle (1993), noted Σx. The abstraction operator is associated with a subordinate DRS, and returns the set of all individuals (if there is any) satisfying this DRS. Although my representation of *more than n* is very close to Kamp & Reyle's (1993: 455), it differs from theirs on two points: (i) I propose this representation only for the *more than* paradigm, not for the *at least* paradigm; (ii) I use only one RD, Σx, not two (Σ/η).

Using this notation, the following DRS can be proposed as a correct semantic representation for *plus de deux* ('more than two'). I repeat the representation of the bare numeral in the table on the left-hand side:

| *Deux étudiants ont appelé.* 'Two students called.' X 　student (X) 　called (X) 　|X| = 2	*Plus de deux étudiants ont appelé.* 'More than two students called.' Σx 　Σx:　x 　　　student (x) 　　　called (x) 　|Σx| > 2

Intuitively, the proposed representation for *more than n* (i) states that the set satisfying the intersection has more than two members; (ii) *introduces* this maximal set into the representation, making it available for anaphoric binding and apposition.

Discourse Referents of type Σ show some peculiarities, as compared to the standard DRs, that is to say atomic and plural DRs:

(i) any maximal set is unique;

(ii) there is no claim (in general) that such a set, built by abstraction on properties, exists.

The first property means that any truthful embedding will project a given Σ on the same set of individuals, and the second property is required by the decreasing operators like *less than n.*

(26)　*Moins de deux étudiants ont appelé.*
　　　'Less than two students called.'

Σx
　Σx : x
　　　student (x)
　　　called (x)

　\|Σx\| < 2
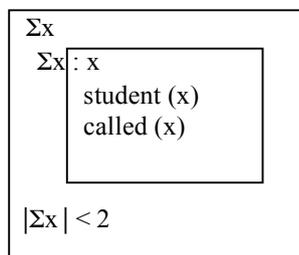
In (26), the presence of a DR at the top level does not imply that the sentence is true iff we can assign individuals of the Model to this DR. The sentence means that Σx, if not empty, has no more than two members.

From the representation assigned to such expressions it is possible to derive some pragmatic constraints on their use. These expressions are analyzed as comparisons: the cardinal of the maximal set is compared to a number. As with any comparison, the speaker should have a reason to evaluate a cardinal by comparison to this particular number without giving the cardinality of the set: any comparison must have some motivation. Here, one can imagine at least two good reasons why the speaker might want to compare the cardinal of Σx with a/this particular number without giving the cardinality of the set:

(i) Because, in the context, there is a statement or an expectation that the cardinality is *n*. Consider, for instance, cases in which *n* is a threshold. If such a standard of comparison is provided in the context, it is even possible to give the comparison *and* the cardinality of the set:

(27)   *Plus de cinq personnes sont venues. Elles étaient en réalité huit.*
       'More than five people came. Actually eight did.'

(ii) Because *n* is a round number:

(28)   *Il y a plus de cent inscrits dans ce groupe.*
       'There are more than one hundred registered people in this group.'

If none of these conditions holds, it is likely that the sentence will be odd, as (28), for instance, is:

(29)   *Il y avait plus de 187 personnes à la réunion.*
       'There were more than 187 people at the meeting.'

Since 187 is not a round number, any speaker accepting (29) will do so because she thinks condition (i) holds, 187 being, for some reason, a good standard of comparison or the expected number.

The satisfaction of one of those conditions answers the question: why do you give this comparison without *giving* the cardinal itself, if you get it?

Another question is: how can you be sure that this comparison is correct without *knowing* the cardinal of the actual set itself? Many situations fulfil this condition. Suppose you begin to count, and then, for some reason, stop at *n*, although you are aware that there are some other items remaining to be counted. It is then natural to state: "All I can say for sure is that there are more than *n* Xs." Another situation is the following: you know, by counting, that a given set A has the cardinality *n*, and, by a rough comparison, that another set B is smaller. It is then natural to say that B has less than *n* elements.

Another typical answer to the question is: it is impossible that the actual set be bigger/smaller, because *n* is a threshold. Consider, for instance: "John has a driving license. He is more than 18, then."

There are also obvious constraints on anaphora and apposition which derive from the special nature of Σ: for instance, *less than n* can be satisfied in models in which there are zero, one, or more individuals of the specified kind; this makes the use of an anaphoric pronoun awkward, because it might be the case that there is no corresponding referent at all, and, in cases where there is, one cannot decide if it is atomic or plural. As regards apposition, it is plausible that some speakers are reluctant to interpret a list as the exhaustive enumeration of Σx because the first part of the sentence does not give the cardinality of the introduced set. In my view, the same problem arises for vague plural indefinites:

(30)   *J'ai lu des livres: A, B, and C.*
       'I read some books: A, B, and C.'

For such sentences, it is not clear whether the speaker gives a sample, or gives the entire list. This is probably why some speakers do not like apposition to numerical comparatives.

To sum up, the specific nature of the postulated set Σ can explain why sentences with numerical comparatives have a restricted dynamic potential.

4. Set Comparators : *au moins* ('at least')*, au plus* ('at most')

There are some differences between set comparators and numerical comparatives:
(i) Set comparators are floating expressions:

(31)　a.　*Au moins deux personnes sont venues.*
　　　　　'At least two people came.'
　　　b.　*Deux personnes au moins sont venues.*
　　　　　'Two people at least came.'
　　　c.　*Deux personnes sont venues, au moins.*
　　　　　'Two people came, at least.'

(ii) Set comparators combine with cardinals, but also with definites:

(32)　*Il a invité au moins ses parents et ses frères.*
　　　　'He invited at least his parents and his brothers.'

(iii) As emphasized by Krifka (1999), they combine with nominal predicates denoting degrees on a scale.

(33)　*Si cette dame est flic, elle est au moins générale.* (B. Lapointe)
　　　　'If this woman is a cop, she is at least a general.'

I will leave aside, in this paper, the uses of these expressions as discourse particles exemplified in (34):

(34)　*Mais, au moins, le travail était fait.*
　　　　'But, at least, the work was done.'

This means that I will concentrate on expressions that have scope over an NP.
　　Kadmon (1987) was the first to note that expressions like *at least* provide two interpretations for a pronoun: a reference to the maximal set and a reference to a set of (exactly) *n* elements. One might add that this extends to definite and demonstrative anaphoric NPs:

(35)　*Au moins deux personnes sont passées ici. Ces deux personnes ont laissé leur trace.*
　　　　'At least two people came here. These two people left their footprints.'

In (35), using the results of the argument given in section 1, one must conclude that the first sentence introduces a set of *exactly two* elements. But the same first sentence can provide another interpretation, illustrated by (36):

(36)　*Au moins deux personnes sont passées ici. Ces personnes, dont nous n'arrivons pas à déterminer le nombre exact (deux seulement, trois, quatre, etc.), ont fait du feu.*
　　　　'At least two people came here. These people (we cannot state for sure how many actually came, only two, three, four, etc.) made a fire.'

In (36), as made explicit by the parenthetical comment, the demonstrative refers to the maximal set of individuals satisfying the conditions expressed by the first sentence.

Data involving apposition to *at least* expressions confirm this:

(37)   *Au moins deux personnes sont passées ici: Jean et Pierre.*
          'At least two people came here: Jean and Pierre.'

In (36), the appositive refers to a set of exactly two persons. This expression usually comes with falling intonation suggesting that it is the exhaustive enumeration of some set. In the logic of the present analysis, we are led to conclude that a discourse referent for this set is introduced in the previous sentence. But sentences like (37) are acceptable as well:

(38)   *Au moins deux personnes sont passées ici: Jean, Pierre, Nicole…*
          'At least two people came here : Jean, Pierre, Nicole…'

The intonation is usually rising, and the sentence suggests then that the list is not finished.

Sentence (37) is perfectly natural for all speakers; (38) is sometimes found less natural, but it is very often accepted. It is also possible to find a list of more than *n* elements, with a *et* prefixed to the last element, as in (38), the list being considered as the exhaustive set of individuals satisfying the predicates of the sentence:

(39)   *Au moins deux personnes sont venues: Jean, Pierre et Nicole.*
          'At least two people came: Jean, Pierre and Nicole.'

From these observations, we can draw the following conclusions:
(i) *at least n* can introduce into the representation a set of (exactly) *n* elements (like numerals);
(ii) *at least n* can introduce into the representation the maximal set (like numerical comparatives).

The most intriguing point is the one that is supported by the strongest empirical data, namely the fact that *at least n* introduces a set of exactly *n* elements (see (35) and (37)). There are two solutions for accommodating the accessibility of these two sets:
A.   The expression *at least n* is ambiguous. Since a lexical ambiguity is not likely, *at least* must be syntactically ambiguous. This is the view adopted by Kadmon (1987), who postulates that *at least* can be either the modifier of the numeral determiner giving a complex determiner *at-least-n*, or an expression taking a whole NP in its scope (*at least (n Ns))*. If *at least* is analyzed as a complex determiner, the sentence introduces the maximal set; if *at least* is conceived of as an operator taking scope over an NP prefixed by a numeral, the latter NP introduces a set of exactly *n* members, just as the bare numeral does in isolation.
B.   The fact that two sets are relevant for the semantic representation of *at least* is not to do with ambiguity; it is simply part and parcel of the semantics of the lexical expression: this expression introduces two sets abstracted over the syntactic environment, and states that a given relation holds between these sets. In this view, every occurrence of the expression makes these two sets accessible.

The A approach raises issues that I cannot take up here. They are discussed in detail in Corblin (to appear). In the present paper, I explore the B approach, i.e. the idea that *at least/at most* introduce two sets, the maximal set Σx and a set of cardinality *n,* and compare the cardinality of these sets.

On the basis of what we have so far assumed for *n* and *more than n*, we can represent the semantics of *at least* in the following way:

		X, Σx
X student (X) called(X) $\lvert X \rvert = 2$	Σx Σ x: | x student (x) called (x) $\lvert \Sigma x \rvert > 2$	Σx: | x student (x) called (x) student (X) called (X) $\lvert X \rvert = 2$ $\lvert \Sigma x \rvert \geq \lvert X \rvert$
Deux étudiants ont appelé. 'Two students called.'	*Plus de deux étudiants ont appelé.* 'More than two students called.'	*Au moins deux étudiants ont appelé.* 'At least two students called.'

The presence of two sets in the representation may seem to offer a chance to accommodate the dynamic properties highlighted above: for instance, in (40), the expression *Pierre et Jean* is an exhaustive enumeration of the set X (a set of exactly *n* members):

(40)　　*Deux étudiants au moins ont appelé: Pierre et Jean.*
　　　　'Two students at least called: Pierre and Jean.'
　　　　X = {Pierre, Jean}

In (41), by contrast, the expression *Pierre, Jean et Marc* is an enumeration of the maximal set Σx:

(41)　　*Deux étudiants au moins ont appelé: Pierre, Jean et Marc.*
　　　　'Two students at least called: Pierre, Jean, and Marc.'
　　　　Σx = {Pierre, Jean, Marc}

But the first difficulty is to state how these sets are constrained. The postulated representation for *at least* given above is awkward: it states that *there is* a set of *n* elements satisfying a given set of conditions S, *and* that the maximal set satisfying S contains *n* elements or more than *n*. Note, however, that if one considers simple sentences conveying separately these two pieces of information, it is impossible to connect them with the conjunction *et:*

(42)　　*Il a écrit deux livres et * il a écrit deux livres ou plus de deux livres.*
　　　　'He wrote two books *and he wrote two books or more than two.'

The only possible combinations are illustrated in (43) and (44):

(43)   *Il a écrit deux livres ou plus de deux livres.*
       'He wrote two books or more than two books.'

(44)   *Il a écrit deux livres et peut être même plus de deux.*
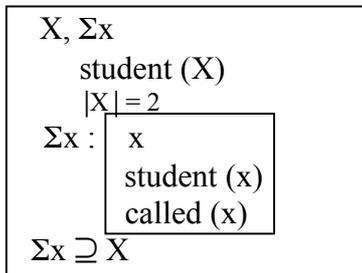   'He wrote two books and maybe even more than two.'

Another problem is related to the formulation of the comparison itself. In the provisional representation above, the comparison is between the cardinality of two sets. It is easy to show that such a representation would not apply to sentences in which *at least* has scope over a definite NP, as in (32). What (32) means is that the maximal set of invited people will *include* the set denoted by the NP *his parents and his brothers*.

If one wants to retain the two-set analysis and deal with these two problems, a solution emerges, which is as follows:
1. *At least* sentences introduce two sets into the representation.
2. One of these sets is Σx, the maximal set of individuals satisfying the conditions expressed in the sentence (except cardinality).
3. The second set, X, is constrained by the sole NP in the scope of *at least.*
4. *At least* expresses the set-theoretic relation $\Sigma x \supseteq X$.

A semantic representation including these features is given in (45):

(45)   *Au moins deux étudiants ont appelé.*
       'At least two students called.'

$$
\boxed{
\begin{array}{l}
\text{X, } \Sigma x \\
\quad \text{student (X)} \\
\quad |X| = 2 \\
\Sigma x : \boxed{
\begin{array}{l}
\text{x} \\
\text{student (x)} \\
\text{called (x)}
\end{array}} \\
\Sigma x \supseteq X
\end{array}}
$$

This representation provides two sources for anaphora and apposition, namely Σx and X. If X receives a specific interpretation, the appositive enumerates the whole set; X can be used without the speaker having a specific set in mind and it amounts, then, to a mere cardinality specification of Σx.

At least n and *n* are verified in the same models, although their commitments are different. The *at least* sentence deals with Σx, and specifies its extension by means of a disjunction ($\supseteq$), whereas, in the *n* sentence, the speaker commits herself to no more than the existence of a set of *n* satisfiers.

It is probably this difference which motivates the thesis that "[at least/at most] modifiers express modal meaning" (Geurts & Nouwen 2005), a position I adopted in the first presentation of this material. I am now less confident that this modal flavor should be considered to be part of the meaning. Although a full discussion is far beyond the scope of this paper, a few comments may be in order.

The modal component one might want to consider, states that it is *possible* that more than *n* elements verify the sentence. It seems that such a commitment could be

seen as a pragmatic inference derived from the assertion of the disjunction $\Sigma x \supseteq X$, not as a part of the meaning proper, and this is a line of thinking I will adopt in the rest of this paper.

The representation in (45) mirrors the following intuition about *at least* sentences: it introduces a set of exactly *n* Xs, and states that this set of Xs is a subset of the maximal set of Xs verifying the predicates of the sentence.

Dynamic data regarding anaphora and apposition show that a two-set analysis is needed for *au plus*. The *exactly n* set is needed, as illustrated by examples like (46):
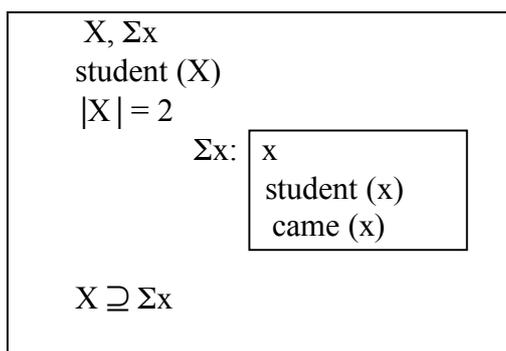
(46) *Deux personnes au plus sont venues ici: Jean et Marc.*
      'At most two people came here: Jean and Marc.'

The meaning of (46) is roughly the following: the set of people who came is empty or included in the set {Jean, Marc}. It is difficult, then, to accommodate (46) without assuming that the sentence introduces a set of *people* of cardinality 2. But this set cannot be a set of people *who came*, since the sentence asserts that the cardinality of this set is zero, one, or two.

This fact can be automatically derived if one makes the rather common assumption that *au plus* belongs to the same semantic category as *au moins*, namely that it takes two arguments, one of them being the set of individuals constrained by the NP it modifies, the other one being the maximal set Σx. The difference between the two items is just that, the meaning of *au plus* being what it is, the relation that is asserted between the two sets is : $X \supseteq \Sigma x$.

In what follows, I provide a tentative representation of (47) which can serve as a template for the representation of all *at most* sentences. In designing this representation, I wish to satisfy the following requirements: (i) preserving the minimal constraint of empirical adequacy for truth conditions and dynamic properties; (ii) preserving the intuition that *at most* and *at least* are related expressions.

(47) *Deux étudiants au plus sont venues.*
      'At most two students came.'

```
┌─────────────────────────────────┐
│  X, Σx                          │
│  student (X)                    │
│  |X| = 2                        │
│          Σx: ┌──────────────┐   │
│              │ x            │   │
│              │ student (x)  │   │
│              │ came (x)     │   │
│              └──────────────┘   │
│                                 │
│  X ⊇ Σx                         │
└─────────────────────────────────┘
```

As said previously, such a representation does not claim that Σx exists: discourse referents of type Σx, although located at the top-level of the DRS, do not assert the existence of the set. The only strong existence claim associated with (47) is the (very weak) claim that there is a set X of two students.

Note that there is a typical circumstance in which we use sentences of this kind: if we know how many individuals of category X there are in a given domain, say *n*, it

is possible to assert correctly, for any property *P, At most n Xs P*. If, for instance, there are three men in a room, *At most three men P* is a tautology for any P.

The representation in (47) correctly predicts the facts for apposition in (48): once again, it suffices to admit that the appended list is interpreted as the exhaustive enumeration of a set introduced in the first part of the sentence, namely the set *X*:

(48)   *Deux étudiants au plus ont appelé: Pierre et Jean.*
'Two students at most called: Pierre and Jean.'
X={Pierre, Jean}

The commitments of (48) are: (i) Pierre and Jean are students; (ii) the set {Pierre, Jean} is a superset of the set of students calling if there are any. This predicts that a sentence like (49) is not interpretable:

(49)   *Deux étudiants au plus ont appelé: Pierre, Jean et Marc.*
'Two students at most called: Pierre, Jean and Marc.'

This representation predicts that two sets are made accessible for definite anaphora: X, a set of *n* Xs, and Σx, the maximal, possibly empty, intersection set, which in any case is of cardinality *n* or smaller than *n*.

A sentence like (49) is not interpretable, because the appended list cannot be interpreted as the enumeration of X (the cardinal of the appended set is not 2), and cannot be interpreted as the enumeration of Σx because this set is smaller than 2.

According to the view advocated in this paper, the quantificational role of *at least/at most* can be described as follows: these expressions work on the basis of a 'witness set' X of cardinality *n* the existence of which is asserted and which is provided by the NP in the scope of the expression; they assert a set-theoretic relation between this set and the maximal set of individuals verifying the conditions expressed in the sentence. This makes these expressions different from numerical comparatives, which compare the cardinality of the (maximal) intersection set to a number (see section 2). The witness set can be specific, the way the corresponding set introduced by a numeral can be. In this case, the speaker has a specific set of *n* individuals in mind as a witness set, which can be, for instance, enumerated in an appositive. It can also be non-specific, as for the corresponding numeral, and then, identification by apposition is impossible.

Again, it is possible to derive modal inferences from a representation so constrained. In stating that the members of Σx, if there are any, belong to a given witness set, the speaker appears to be committing herself to the following propositions:
- It is possible for any member of the witness set to belong to Σx;
- It is impossible that other individuals belong to Σx.

A nice feature of this solution is that it provides a perfect analogy between the semantics of *at least* and *at most*, which is not the case in my previous proposal in Corblin (to appear).[5]

Note that it can also explain nicely why *at most* and *at least* can be conjoined, and what happens when they are conjoined:

(50)   Three students called, at least and at most.

The representation will be, in short: X, Σx: $|X| = 3 \wedge \Sigma x \supseteq X \wedge X \supseteq \Sigma x$. The sentence states, in a rather complicated way, that there is a witness set containing three students, and that the maximal set *Σx* is this set.

Several examples discussed in Corblin (to appear) even show that the witness set can contain entities which do not satisfy the descriptive content of the NP modified by *at most*. Consider the following example:

(51)　*Il y a au plus deux solutions.*
　　　'There are at most two solutions.'

The interpretation of (51) cannot be: there is a set of two solutions, and the maximal set of solutions contains two elements or less than two. What does the sentence mean? Roughly the following: there is a set of two 'things' such that the maximal number of solutions, if there are any, is a subset of this set. The necessity for this set of 'things' is illustrated by sentences like (52):

(52)　*Il y a au plus deux solutions: combattre, ou partir.*
　　　'There are at most two solutions: fighting, or leaving.'

The utterer of (52) is committed to the statement that fighting and leaving are 'possible' solutions, and that if there are solutions, they are in this set.

This is a special example involving existence as main predication, and involving an NP denoting entities (*solution*s) which may not exist. It should be discussed at greater length when considering the construction algorithm required for providing the representation we need. In the standard cases, the general form of the algorithm will be roughly as follows:
1. Build a set X of *n* satisfiers of the NP-denoted predicate to which *at most/at least* is attached;
2. Build a set Σx as the maximal set of Xs satisfying the main predicate;
3. Add the condition stating either $\Sigma x \supseteq X$ or $X \supseteq \Sigma x$;
　　Examples like (52) would require a slight modification of this algorithm.

5. Summary: A Brief Comparison of Numerals and Paranumerals

I now sum up and compare the main features of the three categories.

(a) *trois étudiants*	(b) *plus de deux étudiants*	(c) *trois étudiants au moins*
'three students'	'more than two students'	'at least three students'
Numerals	*Numerical comparatives*	*Set comparators*

The three types, (a), (b) and (c), are true in the same Models. This is general for count nouns, if it is clear that their domain can only be divided by integers. Only (b) and (c) introduce the maximal intersection set: (b) introduces only the maximal intersection set; (c) introduces the maximal set and a witness set of *n* elements. Numerical comparatives, (b), and set comparators, (c), are 'genuine' quantifiers in the sense that the maximal set is part of the picture. Numerals, (a), do not introduce the maximal set, but only a set of *n* members of the intersection set, the relation of this set to the maximal set being left semantically unspecified.

A brief look at the interaction of the three forms with negation shows many differences. The most striking fact regarding negation is that set comparators (*au moins, au plus*) are incompatible with wide scope negation.

For numerals, it is easy to state the contrast between wide scope negation and narrow scope negation:

- narrow scope negation: the sentence claims that there is a set of the relevant cardinality which does not satisfy the predicate;
- wide scope negation: the sentence claims that there is no set of the relevant cardinality satisfying the predicate.

Compare the following sentences under the wide-scope negation reading:

(53)   *Je n'ai pas bu cinq verres de vin.*
         'I did not drink five glasses of wine.'

(54)   *Je n'ai pas bu plus de quatre verres de vin.*
         'I did not drink more than four glasses of wine.'

(55)   *Je n'ai pas bu au moins cinq verres de vin.*
         'I did not drink at least five glasses of wine.'

All the speakers I asked said that (55) is awkward under the wide-scope negation reading, and cannot be accepted except in an echo context. In contrast, (53) and (54) can be used out of the blue under the wide-scope negation reading. A possible explanation in the light of the present proposal would be that the existence of a witness set is necessary for computing the interpretation of set comparators. If one tries to interpret the set X in the scope of the negation, there would be no claim as to the existence of a witness set and the interpretation would be impossible to construct.

Another interesting piece of data concerns the selectional restrictions of predicates taking the expressions under consideration as arguments. Hackl (2001), in his dissertation on comparative quantifiers (our 'numerical comparatives'), points to a puzzle related to the present study. Hackl observes the following contrast between two sentences which should be equivalent in virtue of the equivalence *more than n/ at least n+1* for entities counted with integers:

(56)   ?John separated more than one animal.

(57)   John separated at least two animals.

Roughly speaking, the problem is that, although (56) takes *one* as a part of a complex expression 'corresponding' to a plurality (two, three, or more), it appears as if the only relevant feature for selection were the offending singular 'one', exactly as in **John separated one animal*.

What does the present theory have to say about this problem? At first glance, it predicts that, for *at least n*, *n* should satisfy the selectional requirement exactly as a bare numeral should do. The reason is that we have, so to speak, a numeral interpretation 'within' the representation of the complex expression. Prediction (58) is borne out, and illustrated by (59):

(58)   If *n* violates a selectional restriction, *at least n* violates it too.

(59)   John separated *one animal → John separated *at least one animal.

But it is fair to say that our approach does not expect any problem in the *more than* case. The representation contains only the maximal interpretation *Σx,* which stands for a set of a cardinality which, in cases like (56), should satisfy the selectional requirement.

Our own theory predicts that there should be a strong contrast between *John separated at least one animal*, for which the prediction is out, and *John separated more than one animal*, for which the prediction is correct.

It seems to me that there is actually a strong contrast, and that the *more than* cases are not as bad as Hackl suggests; it can even be observed that there are in French some colloquial uses of *plus d'un* ('more than one') as an understatement for *many*. I think that such expressions can be used even if the predicate imposes a plurality constraint on its argument.

(60)   *Ce chronomètre très précis a départagé plus d'un concurrent.*
        'This very precise chronometer has decided between more than one contestant.'
(61)   *Ce surveillant est amené à séparer plus d'un éléve dans une journée.*
        'This supervisor has to separate more than one pupil in a day.'

To my mind, these examples are correct, whereas their *at least* counterparts would be bad.

6. Some Paranumerals Resisting Classification

Other expressions like *exactement* ('exactly'), *à peu près* ('about'), *environ* ('about'), *en tout* ('in all'), fall under the working definition of the descriptive concept 'paranumeral' given at the beginning of this paper. I now consider each of these expressions in turn to see whether it falls easily into one of the two categories considered up to now.

6.1 Exactement ('exactly')

It is difficult to state whether *exactement* should be classified together with *more than n* or with *at least n*. The problem is that its lexical meaning removes the difference between the two postulated sets distinguished in the course of this study. It is therefore difficult to use dynamic data to establish which set is made available when *exactement* is used. It should be noted that *exactement* behaves with negation the way *at least n* does: a narrow-scope negation reading is acceptable, as illustrated by (62):

(62)   *Je n'ai pas parcouru exactement deux kilomètres.*
        'I didn't walk exactly two kilometers.'

But it is a weak argument for deciding, and I leave the question open.

6.2 Environ, à peu près ('about')

There is a complication here that makes it difficult to use dynamic data for contrasting our two sets: *environ* and *à peu près* select round numbers. For instance, (63) is strange because *47* is not a round number:

(63) **J'ai à peu près 47 étudiants dans mon cours.*
        'I have about 47 students in my course.'

For this reason, it is difficult to contrast an *exactly n* interpretation with a maximal one. Again, I think that the present discussion does not give any conclusive argument for choosing to place *environ* and *à peu près* together with *more than n* or with *at least n*.

6.3 En tout ('in all')

Although *n exactement* and *n en tout* are very often equivalent, there are good grounds for doubting that they belong to the same category. First, *en tout n* can only be used if *n* is obtained by adding different numbers:

(64) *Pierre mesure 1,80 mètres exactement (*en tout).*[6]
        'Pierre is exactly 1.80 meters tall (in all).'

Second, *en tout n* preserves the possibility of interpreting *n* as a round number, which *exactement* prohibits.

(65) *J'ai cinquante étudiants exactement.*   (False if I have 49 sudents)
        'I have exactly fifty students.'

(66) *J'ai cinquante étudiants en tout.*   (True if I have 49 students under
        'I have fifty students in all.'   the interpretation 'round number' of
                                                 fifty)

So *en tout n* does not belong to the *exactement* paradigm. It is probably another kind of paranumeral, associated with the notion of adding together.

7. Conclusion

The main aim of the paper was to contrast different semantic subclasses among paranumerals and to show that many expressions which are often lumped together should be carefully distinguished.

This study shows that, even though their truth conditions are often the same, numerals, numerical comparatives, and set comparators can be distinguished on the basis of some of their dynamic properties. What numerals introduce into the representation is a set of exactly *n* individuals satisfying the conjunction of the NP and VP constraints and, moreover, the existence of this set is asserted. Numerical comparatives (*more/less than n*), on the other hand, only introduce into the representation the maximal set of individuals Σx satisfying the conjunction of the NP and VP constraints, and compare the cardinality of this set to *n*. As for set

comparators (*at least/at most)*, they introduce two sets into the representation: Σx, and a witness set. This latter set is constrained as a set of n Xs, X being the descriptive content of the NP, and its existence is asserted.

The paper focuses on typical properties and makes suggestions about how these properties can be dealt with within the DRT framework, at least as regards a restricted set of distributions, namely the use of those expressions in construction with NPs containing a numeral.

Many details have been left out in an attempt to bring out the main contrastive features of expressions in the numeral/paranumeral domain.

Among the important points deserving careful attention is the range of constructions allowed for each type of expression. It remains to be explained why those expressions can be regarded as paranumerals (the association with numerals is typical), and why they are not restricted to co-occurrence with numerals. The present study, in other words, would have to be subsumed under a more general theory, one powerful enough to apply to the whole range of distributions, and to explain why numerals are typically part of the distribution. Such a theory is far beyond the scope of this paper, but I would like to suggest, as an opening, a contrast in line with a difference postulated in this paper.

At least/at most cannot combine with degree adjectives:

(67)   *Il est au moins *grand /*froid.*
          'It is at least great/cold.
,

At least/at most can only work if they are attached to a constituent that can be interpreted as identifying a precise measure on a scale. The constraint may be less absolute for comparatives, although it applies strictly to decreasing comparatives:[7]

(68)   *Il est moins que *grand /*froid.*
          'It is less than tall/cold.'

A detailed study of the restrictions would be in order before deciding if this requirement holds only for *at least/at most*, or can be generalized.

The notion of 'witness set' used in the present approach to explain the kind of semantic calculus associated to *at least/at most* would provide a good explanation of why this is so: a witness set works as some sort of yardstick, used for the evaluation of a measurable dimension of the maximal set; this is why an acceptable argument of *at most/at least* must provide a definite point on a scale (a cardinality, or the name of a recognized degree) and not a vague comparison to a standard, as degree adjectives would do. Further work on the *more than* paradigm would introduce, if the requirement can be generalized, a very interesting contrast between *more+than+adj*, which puts a ban on degree adjectives, and *more+adj+than*, which selects a degree adjective. The fundamental contrast to be explored is the contrast between comparing the dimension of something to a measure on the relevant scale, and comparing the location of two entities on a scale.

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Notes

[*] This paper is a version of the talk presented at the conference *Indefinites and Weak Quantifiers* held in January 2005. I owe many thanks to two anonymous referees of this paper. Their comments and demands of justification of the version presented at the conference have played a great role in convincing me that it is possible to provide a semantic representation of *at least/at most* without any appeal to a modal part of the representation (for an opposite view, see Geurts & Nouwen 2005). This explains that the paper presents an analysis of these expressions which is closer to my former analysis (Corblin, to appear) than the version presented at the conference. I read Geurts & Nouwen (2005) when I was finishing the revision of the present paper, which explains why it was not possible to incorporate a discussion of their work. Since my first presentation of this material in the 2002 Nancy workshop *Existence: Semantics and Syntax*, I got very interesting comments from B. Geurts, and I had the opportunity to hear a couple of talks by him on this topic. I am now convinced that a semantics without built-in modality is a better way to deal with paranumerals, and I encourage the reader to read Geurts & Nouwen (2005) for an opposite view. My work on this topic has greatly benefited from discussions with many other people, among others, G. Chierchia, P. Dupuy, O. Matushansky, A. Merin, and V. Stanojevic.

[1] I owe to an anonymous referee of this paper the following example:

Il faudrait plus que Pierre et Jean (la dernière réunion/mon dernier échec) pour me décourager.

'It would take more than Peter and John (the last meeting, my latest failure) to depress me.'

There are many differences in acceptability between *plus que/moins que* and *plus de/moins de* that I will not discuss in this paper.

[2] Merin (2003) provides very strong criticism of this dominant view, and gives very good arguments for returning to the thesis that *n* means 'n', and has no other meaning.

[3] For the sake of simplification, I do not take into account in this paper the contrast between round numbers and others.

[4] A fact discovered by Kadmon (1987).

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[5] The main difference is that, in my previous treatment, the set X in the representation of *at least* was defined as a set of individuals verifying the descriptive content of the NP *and* the predicate (very similar, then, to the interpretation of a bare numeral). Such a choice has two negative consequences: the representation of *at least* is, so to speak, redundant, and the plain extension of this representation, *mutatis mutandis*, to *at most* is impossible. In this slightly different version, these two problems are fixed.

[6] A point made by Pascal Dupuy (2004).

[7] Positive comparatives can be used more freely, although they produce with many degree adjectives a meaning close to *very*: *C'est plus que froid* ('It is more than cold') is interpreted as *It is very cold*.